Mid-term exam in macroeconomics 2007-2008

Exercise 1: growth and saving rate (50% of marks)

It is assumed that the aggregate good of the economy is produced by one representative firm endowed with the production technology: $Y_t = K_t^{\alpha}(N_t)^{1-\alpha}$ with α such that: $0 < \alpha < 1$. Y_t is aggregate output, K_t the total capital stock and N_t is the quantity of labor. There is no technical progress. N_t increases at the constant rate $n : N_{t+1} = (1+n)N_t$.

1. The saving rate of agents is equal to s. The depreciation rate of the capital stock is δ. Write the accumulation equation of the capital stock.
2. The variable k_t is defined as K_t/N_t. Find the equation defining the dynamics of k_t? On a drawing, show that k_t converges toward a stable steady state k^*. Give the expression of k^*. What is the effect of s on k^*?
3. What are the paths of the main variables of the economy in the long run: k_t, K_t, Y_t and $y_t = Y_t/N_t$. What is the effect of an increase of s?
4. From this question, it is assumed that the saving rate of each agent is an *increasing function* of his income. It is assumed that all agents earn the same income y_t. The saving rate is now denoted by a function: $s = \sigma(y_t)$. Can you interpret this assumption?
5. To have simple results, it is assumed that the function σ is such that:

$$\sigma(y) = s_1 \text{ for } y < \bar{y}$$
$$\sigma(y) = s_2 \text{ for } y > \bar{y}$$

with $s_1 < s_2$. Moreover, it is assumed that $\bar{y}$ is such that

$$\left(\frac{s_1}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} < \bar{y} < \left(\frac{s_2}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Represent by a drawing how k_{t+1} depends on k_t. What is the long run value of k_t? Is this long run value independent of the initial value k_0? Compare the results with the one found in question 3.

6. What can you say about the dynamics of k_t in the case

$$\bar{y} < \left(\frac{s_1}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

What can you say if:

$$\left(\frac{s_2}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} < \bar{y}$$

Compare the results with questions 5 and 3.

Exercise 2: consumption with imperfect capital market (50% of marks)

This exercise studies the consumption behavior of an agent living during 3 periods 1, 2 and 3. This consumer has no initial asset. He earns in periods 1 and 3 an income (real) $y > 0$ and he has no income in period 2. He consumes at each period $i = 1, 2$, and 3 a quantity c_i of the aggregate good of the economy. a_1 denotes the amount of asset (or savings) at the end of period 1, and a_2 at the end of period 2. The utility function of the agent is:

$$U(c_1, c_2, c_3) = \ln c_1 + \ln c_2 + \ln c_3$$

1. This question assumes that the agent can borrow or lend on a perfect capital market, with a real interest rate $r = 0$.
1.a Write the budget constraints at each period. Give the expression of the intertemporal budget constraint.
1.b Write the consumer's maximization program. Give the first order conditions of this program. Give the solutions of this program: the expressions of c_1, c_2, c_3, a_1 and a_2.
1.c Represent on a drawing the path of income, consumption and savings with respect to time.
2. In this question, the capital market is imperfect: it is not possible to borrow. Thus, the consumer is constrained at each period by the constraint: $a_i \geq 0$, $i = 1, 2$. The interest rate remains equal to 0.
2.a What are the new first order conditions of the consumer program, taking into account the new constraints?
2.b What is the new solution: the expressions of c_1, c_2, c_3, a_1 and a_2.
2.c Represent on a drawing the path of income, consumption and savings with respect to time. Compare the results with these of question 1.