

Final exam in macroeconomics 2007-2008

Exercise 1: growth and public good (50% of marks)

I. In this part, the production technology of the economy is given by: $Y_t = AK_t$. Y_t is the aggregate output at period t , K_t is the physical capital stock. The depreciation rate of the capital stock is full ($\delta = 1$). A is a constant parameter.

One representative consumer is living in this economy, endowed with the following intertemporal utility function:

$$\sum_{t=0}^{+\infty} \beta^t \ln(c_t)$$

β is positive and < 1 . c_t is the level of consumption at period t .

1. Write the optimization program that define the optimal state of the economy.
2. Write the optimality conditions.
3. Find the growth rate of consumption c_t .
4. Show that K_t increases at the same rate.

II In this part, the production function is $Y_t = AK_t^\alpha G_t^{1-\alpha}$ with $0 < \alpha < 1$. G_t represents an amount of public good that is used in production. G_t is provided by the government, that is able to transform one unit of consumption good into one unit of public good. The new resource constraint of the economy is then:

$$AK_t^\alpha G_t^{1-\alpha} = c_t + G_t + K_{t+1}$$

1. Write the optimization program that define the optimal state of the economy. In this program, c_t and G_t are the command variables and K_t the state variable. Find the new optimal state of this economy. What is the relation between G_t and K_t along the optimal path?

Exercise 2: Growth in an OLG model with a PAYG pension system (50% of marks)

A model with overlapping generations is considered, in which N_t agents are born at each period t . N_t increases at a constant rate n : $N_t = (1 + n)N_{t-1}$. Each agent is living during 2 periods. He supplies one unit of labor during his first period of life, and is retired during the second period. The utility function of a generation t agent depends on his consumptions during the two periods, denoted by c_t and d_{t+1} , and is defined by:

$$U(c_t, d_{t+1}) = (1 - a) \ln c_t + a \ln d_{t+1}$$

The parameter a is such that: $0 < a < 1$. s_t is the amount of savings held by the agent at the end of period t .

The production technology is Cobb-Douglas:

$$Y_t = K_t^\alpha (B_t L_t)^{1-\alpha}$$

K_t is the capital stock, L_t the quantity of labor. B_t represents the technical progress. Its expression will be defined after. There is full depreciation of capital in one period. w_t is the wage in period t , R_t the factor of interest.

1. Write the budget constraints of an agent of generation t , and his intertemporal budget constraint. Find the optimal levels of consumptions and savings.
2. Write the firm program at period t . Give the optimality conditions.
3. It is assumed that the technical progress B_t is defined by $B_t = K_t/N_t$. Can you interpret this assumption? Under this assumption, and using the equilibrium condition on the labor market, give the expressions of w_t and R_t with respect to $k_t = K_t/N_t$.
4. Write the equation expressing the equilibrium of the capital market. Show that k_t increases at a constant rate. Explain the impact of the different parameters on this growth rate.
5. A PAYG pension system is introduced in the economy. Young agents income is taxed at a constant rate τ . This tax finances a pension for old agents. p_t denotes the amount of the pension received by old agents in period t (agents of generation $t - 1$).
 - 5.a Write the new budget constraints and solve the consumer program, find the new amount of savings.
 - 5.b Write the equation ensuring that the budget constraint of the social security system is balanced at each period t .
 - 5.c Find the new expression of the dynamics of k_t . What is the impact of τ on the growth rate of k_t ? How can you interpret this result?