

Chapter 3: exercises

Ex 1: Competitive equilibrium and social optimum

I. In this part, an economy with one representative firm is studied, with the following production function: $Y_t = AK_t(N)^{1-\alpha}$ where $0 < \alpha < 1$. Y_t is the aggregate output at period t , K_t is the physical capital stock and N is the quantity of labor (constant). The depreciation rate of the capital stock is equal to δ . A is a constant parameter.

One representative consumer is living in this economy, endowed with the following intertemporal utility function:

$$\sum_{t=0}^{+\infty} \beta^t \ln(c_t)$$

Find the optimum state of this economy.

II In this part, the production function is $Y_t = AK_t^\alpha(B_tN)^{1-\alpha}$ where $0 < \alpha < 1$. B_t represents the technical progress. Its expression will be defined in the sequel.

1. Write the equations that must be satisfied along a competitive equilibrium of this economy.
2. It is assumed that the technical progress B_t is defined by $B_t = K_t$. This assumption is taken from Romer (1986). K_t is used as a proxy variable for the knowledge B_t acquired in the economy at period t . B_t is an externality for the firm, and is taken *as given*.

Under this assumption, what are the features of the competitive equilibrium ?

3. Find the optimum state of this economy. What is the difference with the competitive equilibrium ? Why does this difference exist ? Can you imagine some policy allowing the competitive economy to reach the optimum state ?