Chapter 1: exercises

Ex 1: Catching up

In 1995, average GDP per capita is equal to $y_d = 25000\$$ for developed countries, and $y_u = 1200\$$ for under-developed ones. The average rate of growth are respectively $g_d = 3\%$ and $g_u = 2\%$.

1) Considering the average growth rate of GDP per capita in under-developed countries, what time would be necessary to reach the average level in 1995 of developed countries ?

2) In 1995, Malaysia had a GDP per capital equal to 4000\$ and a growth rate of $g = 6\%$. Assuming that this growth rate remains constant in the future, how long would it take for Malaysia to catch up the high income countries? How long would it take for $g = 4\%$?

3) What should be the growth rate in under-developed countries to catch up developed one in 20 years ?

Ex 2

We consider an economy with one representative firm endowed with the following production function: $Y = K^{\alpha}H^{\beta}(AN)^{1-\alpha-\beta}$ where α and β are positive constants such that: $\alpha + \beta < 1$. Y is the aggregate output, K is the physical capital stock and H the human capital stock. N is the quantity of labor and A a technical progress. N and A are growing at a constant rate: $A_{t+1}/A_t = 1 + a$ and $N_{t+1}/N_t = 1 + n$.

1. What can you say about the production function ?

2. We assume that a share s_k of the output is invested in physical capital, and a share s_h is invested in human capital. The depreciation rates of the two stocks are equal to the same value δ. What is the accumulation equation of each stock?

3. We introduce the following variables:

$$h_t = \frac{H_t}{A_t N_t} \text{ and } k_t = \frac{K_t}{A_t N_t}$$

What is the dynamics of these two variables?

4. What are the (constant) long run values of h and k ? What is the impact of each parameter of the model on these variables ? What are the long run dynamics of H_t, K_t and Y_t ?

5. We now consider the particular case $\alpha + \beta = 1$. What can you say on the long run properties of this model ? What is different with respect to the preceding model ?

6. We consider two countries which are endowed with different values for saving rates s_k and s_h. Compare the long run growth of these two economies using successively the first ($\alpha + \beta < 1$) and the second ($\alpha + \beta = 1$) model ?

Ex 3

Consider the Solow's model with production given by the following function:

$$Y = F(K, AN) = \min\left(\frac{K}{v}, \frac{AN}{u}\right)$$

with a end b two technical coefficients. Factors are complements in the production process. Y is the aggregate output, K is the physical capital stock, N is the quantity of labor and A a technical progress. N and A are growing at a constant rate: $A_{t+1}/A_t = 1 + a$ and $N_{t+1}/N_t = 1 + n$. What can you say on the growth process in the 3 cases:

$$\left(1 - \delta + \frac{s}{v}\right) \begin{array}{c} > \\ = \\ < \end{array} (1 + n)(1 + a)$$