Exercises on chapter 2

Exercise 1: consumption with imperfect capital market

This exercise studies the consumption behavior of an agent living during 2 periods 1 and 2. This consumer has no initial asset. He earns at each period $i = 1, 2$ an income (real) Y_i, and he consumes at each period $i = 1, 2$ a quantity C_i of the only good of the economy. S denotes the amount of asset (or savings) that is saved at the end of period 1. The utility function of the agent is:

$$U(C_1, C_2) = (1 - a) \ln C_1 + a \ln C_2$$

1. This question assumes that the agent can borrow or lend on a perfect capital market, with a real interest rate r.
1.a Write the budget constraints at each period. Give the expression of the intertemporal budget constraint. Draw this intertemporal constraint in the plane (C_1, C_2).
1.b Write the consumer's maximization program. Give the solutions of this program: the expressions of C_1, C_2 and S. What is the impact of r on these three variables ?
1.c Show that there exists a value of the interest rate denoted by $\widehat{r}$, such that, S is positive for $r > \widehat{r}$, and S is negative for $r < \widehat{r}$.
2. In this question, the capital market is imperfect: the borrowing rate is $\bar{r}$ while the lending rate is $\underline{r}$, with $\underline{r} < \bar{r}$. Find the new budget constraints in the two possible cases and the intertemporal budget constraint. Draw this intertemporal constraint in the plane (C_1, C_2).
2.b Assuming that the agent is borrower ($S < 0$), give the value of his savings S (without any new calculation). Find the condition on $\bar{r}$ ensuring that the ex-ante assumption ($S < 0$) is satisfied ex-post.
2.c Assuming that the agent is lender ($S > 0$), give the value of his savings S (without any new calculation). Find the condition on $\underline{r}$ ensuring that the ex-ante assumption ($S > 0$) is satisfied ex-post.
2.d Give the saving behavior of the agent in the three cases: $\widehat{r} > \bar{r}$, $\widehat{r} < \underline{r}$ and $\underline{r} < \widehat{r} < \bar{r}$.

Draw these results in the plane (C_1, C_2). What is the consequence of imperfect capital markets on savings?
3. In this question, the capital market is imperfect: it is not possible to borrow ($S \geq 0$). The interest rate is r for savings. Draw the new intertemporal budget constraint. Find the optimal values of C_1, C_2 and S. What is the impact of an increase of r, or of an increase of Y_1 on these variables?

Exercise 2

An economy is living during two periods, $i = 1, 2$. This economy is populated by N identical agents that are living during the two periods 1 and 2. They have no initial asset. They earn at each period $i = 1, 2$ an income (real) y_i, and consume at each period $i = 1, 2$ a quantity c_i of the only good of the economy. s denotes the amount of asset (or savings) that is saved by each agent at the end of period 1. The utility function of the agent is:

$$U(c_1, c_2) = (1 - a) \ln c_1 + a \ln c_2$$

In this economy also exists a government that levies taxes in both periods. Each agent must pay an amount of taxe τ_i at period i. These taxes finance public spendings: G_1 in period 1 and G_2 in period 2. All agents can borrow or lend on a perfect capital market, with a real interest rate r.

1. Write the intertemporal budget constraints of each agent, and the one of the government. Making the sum of all these constraints, how can you interpret the result?
2. Write the consumer's maximization program. Give the expressions of c_1, c_2 and s.
3. In this question, it is assumed that the government lets G_1 and G_2 constant, but it reduces the amount of τ_1 by 1. The amount of taxes in period 1 is now $\tau_1 - 1$ instead of τ_1. What is the new value of τ_2?
4. What is the impact of this change in the path of taxes on c_1, c_2 and s?

Exercise 3

Consider a consumer who is living during periods $t = 0, ...T$. We denote by c_t his consumption at period t, $y_t = y$ his income (constant), and a_t the amount of asset that he holds at the end of period $t - 1$. a_0 is the initial amount of the asset ($a_0 \geq 0$). The agent can borrow or lend on a perfect financial market on which the interest rate is constant and equal to: r. The budget constraint at each period can be written:

$$a_{t+1} = (1 + r)a_t + y_t - c_t$$

The intertemporal utility is defined as:

$$\sum_{t=0}^{T} \beta^t \ln (c_t) \quad \text{with} \quad 0 < \beta < 1$$

1) Consumer behavior without constraint

Find the first order conditions (Euler's conditions) of the consumer program. Give the expression of the intertemporal budget constraint. How is the consumption profile ? Make a drawing representing the evolution of c_t and a_t with respect to t. Show that 3 cases appear according to the values of β and r. The discussion will be made in using the variable δ such that $\beta = 1/(1 + \delta)$.

2) The constrained behavior

In this question, financial markets are no more perfect: it is impossible to borrow. Therefore, consumer's behavior is constrained at each period by the inequality: $a_t \geq 0$. Show that first order conditions now become:

$$-u'(c_t) + \beta(1+r)u'(c_{t+1}) \leq 0$$

with a strict inequality if $a_t = 0$ and an equality if $a_t > 0$.
Show that the consumer cannot be constrained in the case $r \geq \delta$. In the converse case, give the method to solve the consumer program (without solving !). On a drawing, represent the consumption profile and compare it with the profile obtained in the first question.

Exercise 4

We consider a consumer living during the periods $t = 1, ...T$. His instantaneous utility depends both on his current consumption c_t at period t, and on the amount of real balances that he holds m_t. We define real balances in period t by: $m_t = M_t/P_{t-1}$, with M_t the quantity of money held in t and P_{t-1} the price level index in period $t-1$. The consumer can save between two periods under two forms. He can buy some amount (nominal) A_t of an asset or he can keep some amount of money M_t. The asset allows to earn a nominal interest rate equal to i. Money does not provide an interest rate and its nominal return is zero. The inflation rate between two periods is constant and equal to π:

$$\pi = \frac{P_{t+1} - P_t}{P_t}$$

Therefore, the real interest rate is equal to r such that:

$$1 + r = \frac{1+i}{1+\pi}$$

Finally, the agent earns a nominal income W_t at each period t, and we assume that the real value of this income remains constant: $W_t/P_t = \omega$.

The intertemporal utility of the consumer is defined as:

$$\sum_{t=1}^{T} \beta^t u(c_t, m_t) \quad \text{with} \quad 0 < \beta < 1 \quad \text{and} \quad u(c,m) = \ln(c) + \ln(m)$$

A) First part

1) How can you interpret such an intertemporal utility function ?
2) Show that at period t, the budget constraint can be written:

$$A_{t+1} + P_t c_t + M_{t+1} = (1+i)A_t + M_t + W_t$$

3) We denote by $a_t = A_t/P_{t-1}$ the real value of the asset and by $x_t = a_t + m_t$ the real value of total savings. Show that the budget constraint of the

consumer can be written (with a standard approximation) with real variables as:

$$x_{t+1} = (1+r)x_t - c_t - (r+\pi)m_t + \omega$$

How can you interpret the expression $r + \pi$?

For the period $t = 0$, we assume that the total initial endowment x_0 is given, but that m_0 and a_0 can be chosen freely in $t = 0$. Why ?

What is the terminal value of total wealth x_{T+1}?

4) What are the optimality conditions of the consumer program ?
5) Find c_t and m_t. On a drawing, represent the consumption and real balances profiles with respect to the values of r and β.

B) second part

In this part, we now assume that the instantaneous utility function u has the following form:

$$u(c, m) = \ln(\min(c, m))$$

1) Can you explain this assumption on the utility function ?
2) How is it possible to write the budget constraint ?
3) Solve the consumer program and find c_t and m_t. What are the differences with respect to the first part ? Discuss the influence of the parameters r and π on money holding.