

Chapter 2: Consumption behavior

1 Introduction

Aim of this chapter: to define a macroeconomic consumption function.

In Solow's model, the aggregate consumption function is

$$C = (1 - s)Y$$

cf. Keynes consumption function:

$$C = cY + C_0$$

Drawbacks of this formulation:

No micro-foundations (consumption should depend on present and future incomes, and on inherited wealth)

Empirical problems in estimating c and C_0 .

2 The life cycle model of consumption (Modigliani)

2.1 Basic assumptions

Consider one agent, living from $t = 0$ to $t = T$. T is known.

agent's income (in real terms): y_t .

One aggregate consumption good. c_t is consumed in t .

A perfect capital market with a real rate of interest r_t between $t - 1$ and t .

Budget constraints: a_t is the quantity of asset held by the agent at the end of period $t - 1$.

$$a_{t+1} = (1 + r_t)a_t + y_t - c_t$$

$a_{T+1} \geq 0$ (no debt at the death period). It is straightforward that the optimal choice will be $a_{T+1} = 0$

Notation:

$$\rho_t = \frac{1}{\prod_{i=0}^t (1 + r_i)}$$

The intertemporal budget constraint:

$$\sum_{t=0}^T \rho_t c_t \leq a_0 + h_0$$

with $h_0 = \sum_{t=0}^T \rho_t y_t$

h_0 is the human wealth in 0.

Total wealth is $W_0 = a_0 + h_0$.

Consumer intertemporal utility

$$\sum_{t=0}^T \beta^t u(c_t)$$

is additively separable.

u is the instantaneous utility function. $u' > 0$, $u'' < 0$.

$\beta = 1/(1 + \delta)$, δ the discount rate (= rate of time preference).

2.2 Consumer program

$$\begin{aligned} & \max \sum_{t=0}^T \beta^t u(c_t) \\ \text{s. t. } & \sum_{t=0}^T \rho_t c_t \leq a_0 + h_0 = W_0 \\ & a_0 \text{ given} \end{aligned}$$

FOC:

$$u'(c_t) = \beta (1 + r_{t+1}) u'(c_{t+1})$$

(Euler-Keynes-Ramsey equation)

Particular case:

$$u(c) = \frac{c^{1-1/\sigma}}{1-1/\sigma}$$

$\sigma \in (0, +\infty)$ is the intertemporal elasticity of substitution.

Cases $\sigma \rightarrow 0$, $\sigma \rightarrow \infty$.

Solution:

$$c_t = \frac{\beta^{\sigma t} \rho_t^{-\sigma} W_0}{\sum_{t=0}^T \beta^{\sigma t} \rho_t^{1-\sigma}}$$

Case $r_t = r$.

Case $\sigma = 1, \sigma = 0, \sigma = +\infty$.

Case $r_t = \delta$.

3 The representative consumer in macroeconomic models

A macroeconomic function for consumption can be obtained:

either in aggregating consumptions of different agents from different generations (overlapping generation model)

or in assuming one representative agent with an infinite life.

We study here the second solution.

3.1 The representative agent/family

We consider that there exists a representative agent/family in the economy. The population within the family grows at a constant rate n : $N_{t+1} = (1 + n)N_t$.

Intertemporal utility:

$$\sum_{t=0}^{+\infty} \beta^t u(c_t)$$

(or $\beta = \beta'(1 + n)$).

The dynamics of current wealth a_t of the representative infinite-lived consumer is :

$$a_{t+1} = \frac{1}{1 + n} [(1 + r_t)a_t + y_t - c_t]$$

or

$$A_{t+1} = (1 + r_t)A_t + N_t y_t - N_t c_t$$

with $A_t = N_t a_t$ the total asset of the family in t .

How to write the intertemporal budget constraint ?

$$\lim_{t \rightarrow \infty} \rho_t A_{t+1} \geq 0$$

In fact, 3 equivalent ways:

1.

$$\lim_{t \rightarrow \infty} \rho_t A_{t+1} \geq 0$$

2.

$$\sum_{t=0}^{+\infty} \rho_t N_t c_t \leq A_0 + \sum_{t=0}^{+\infty} \rho_t N_t y_t$$

3. For all t ,

$$A_t + H_t \geq 0$$

with

$$H_t = \sum_{s=t}^{+\infty} \frac{\rho_s}{\rho_{t-1}} N_s y_s$$

The intertemporal budget constraint can only be defined if the sum $\sum_{t=0}^{+\infty} \rho_t N_t y_t$ is well-defined (convergent).

The first order condition remain the same:

$$u'(c_t) = \beta \frac{1 + r_{t+1}}{1 + n} u'(c_{t+1})$$

and the limit condition is satisfied with an equality:

$$\lim_{t \rightarrow \infty} \rho_t A_{t+1} = 0$$

3.2 Ricardian equivalence

Assume that there is a state in the economy. The state issues public debt B_t (in period t , not $t-1$), makes public expenses G_t , collects non-distortionary taxes (lump-sum taxes) τ_t .

With no uncertainty, debt and capital must have the same return.

at each date, the budget constraint of the state is:

$$B_t = (1 + r_t)B_{t-1} + G_t - N_t\tau_t$$

For the family budget constraint, we can write: $y_t = w_t - \tau_t$, and

$$A_{t+1} = (1 + r_t)A_t + N_t(w_t - \tau_t) - N_t c_t$$

Total asset held by the family is shared between productive capital and public debt:

$$A_t = K_t + B_{t-1}$$

As for the representative consumer $\lim_{t \rightarrow \infty} \rho_t A_{t+1} \geq 0$, we must have

$$\lim_{t \rightarrow \infty} \rho_t (K_{t+1} + B_t) = 0$$

As $K_{t+1} \geq 0$, we have $\lim_{t \rightarrow \infty} \rho_t K_{t+1} \geq 0$, and therefore $\lim_{t \rightarrow \infty} \rho_t B_t \leq 0$.

If we restrict the analysis to the case $B_t \geq 0$, we obtain:

$\lim_{t \rightarrow \infty} \rho_t B_t = 0$, or the intertemporal budget constraint of the state:

$$B_{-1} + \sum_{t=0}^{+\infty} \rho_t G_t = \sum_{t=0}^{+\infty} \rho_t N_t \tau_t$$

Writing the intertemporal constraint of the representative family:

$$\sum_{t=0}^{+\infty} \rho_t N_t c_t = A_0 + \sum_{t=0}^{+\infty} \rho_t N_t (w_t - \tau_t)$$

the sum gives:

$$\sum_{t=0}^{+\infty} \rho_t N_t c_t + \sum_{t=0}^{+\infty} \rho_t G_t = K_0 + \sum_{t=0}^{+\infty} \rho_t N_t w_t$$

This equation means that paths of taxes τ_t and public expenses G_t have no impact on consumption choices. The only term that matters is the sum of actualized public expenses.

Ricardian neutrality: a decrease of τ_t financed by an increase in B_t has no impact on consumption, and only an impact on A_{t+1} that increases to buy the debt.

Government bonds are viewed by the consumers as future taxes.

This result needs strong assumptions: perfect foresights, non distortionary taxes, perfect capital markets...

4 Consumption choice under uncertainty

We assume now that income is uncertain.

For a random variable x_t , we denote by $E_t(x_{t+i})$ the expected value of x_{t+i} conditional on information at time t .

$$E_t(x_{t+i}) = E(x_{t+i} | I_t).$$

An important property of conditional expectations is that, for $1 \leq j < i$:

$$E_t(E_{t+j}(x_{t+i})) = E_t(x_{t+i})$$

We assume that y_t is a random variable.

Examples:

$$y_t = y + \varepsilon_t$$

with ε_t an iid random variable.

$$y_t = \rho y_{t-1} + (1 - \rho)y + \varepsilon_t$$

In the first example, the observation of y_t does not give any information on future realizations y_{t+i} . In the second example, the observation of y_t gives information on future realizations y_{t+i} .

The consumer objective:

$$E_0 \left(\sum_{t=0}^T \beta^t u(c_t) \right)$$

We are looking for closed-loop solutions: a behavior rule of the form: $c_t = C_t(a_t)$.

a_t is the state variable, which depends on all the path of preceding shocks.

c_t is the command variable.

The budget constraint remains the same, with a random income:

$$a_{t+1} = (1 + r_t)a_t + y_t - c_t$$

We assume that the value of y_t is observed before the choice of c_t .

Finally, we assume that r_t is constant: $r_t = r$. We define $R = 1 + r$.

Method for the resolution: the dynamic programming principle = Assume that the problem is solved.

We define the value function:

$$V_t(a_t) = \max E_t \left(\sum_{i=t}^T \beta^{i-t} u(c_i) \right)$$

under the budget constraint

We have the Bellman equation:

$$V_t(a_t) = \max u(c_t) + \beta E_t (V_{t+1}(a_{t+1}))$$

s. t. $a_{t+1} = Ra_t + y_t - c_t$

Foc:

$$u'(c_t) = \beta E_t \left(V'_{t+1}(a_{t+1}) \right)$$

$$u'(c_t) = \beta E_t \left(V'_{t+1}(Ra_t + y_t - c_t) \right)$$

Assume that the function V_{t+1} is known. The last equation implicitly defines the behavior rule: $c_t = C_t(a_t)$.

The derivative of the Bellman equation with respect to a_t for the optimal consumption choice $c_t = C_t(a_t)$ gives the condition:

$$V'_t(a_t) = R\beta E_t \left(V'_{t+1}(a_{t+1}) \right)$$

The 2 last conditions give the Euler equation:

$$u'(c_t) = \beta R E_t \left(u'_{t+1}(c_{t+1}) \right)$$

To obtain an explicit solution, we will assume henceforth:

$$\begin{aligned}\beta R &= 1 \\ u(c) &= ac - \frac{b}{2}c^2\end{aligned}$$

We obtain:

$$c_t = E_t(c_{t+1})$$

c_t follows a random walk. (Hall 78).

$$c_{t+1} = c_t + \eta_{t+1}$$

with $E_t(\eta_{t+1}) = 0$.

The complete resolution is obtained by backward induction:

For $t = T$,

$$c_T = y_T + Ra_T$$

For $t = T - 1$, c_{T-1} is solution of:

$$\begin{aligned}a_T &= Ra_{T-1} + y_{T-1} - c_{T-1} \\ c_{T-1} &= E_{T-1}(c_T) = E_{T-1}(y_T + Ra_T)\end{aligned}$$

or

$$(1 + R)c_{T-1} = E_{T-1}(y_T) + Ry_{T-1} + R^2a_{T-1}$$

By induction:

$$c_t = \frac{E_t(y_T + Ry_{T-1} + \dots + R^{T-t-1}y_{t+1})}{1 + R + R^2 + \dots + R^{T-t}} + \frac{R^{T-t}y_t + R^{T-t+1}a_t}{1 + R + R^2 + \dots + R^{T-t}}$$

Dividing by R^{T-t} :

$$c_t = \frac{y_t + E_t\left(\frac{y_{t+1}}{R} + \dots + \frac{y_{T-1}}{R^{T-t-1}} + \frac{y_T}{R^{T-t}}\right) + Ra_t}{1 + R^{-1} + R^{-2} + \dots + R^{t-T}}$$

If $T \rightarrow +\infty$, we obtain:

$$c_t = ra_t + \frac{r}{1+r} \left(y_t + E_t \left(\sum_{i=1}^{+\infty} \frac{y_{t+i}}{R^i} \right) \right)$$

A theoretical/empirical debate: Does consumption follow a random walk process ?