1 Chapter 1: Economic growth

Reference: Barro and Sala-i-Martin: Economic Growth, Cambridge, Mass. : MIT Press, 1999.

1.1 Empirical evidence

Some stylized facts

Nicholas Kaldor at a 1958 conference provides some stylized facts that growth theory must explain.

Fact 1: A steady growth rate for the real GDP and the real per capita GDP

Some notations/definitions: Y_t, y_t, g_t

$$1 + g_t = \frac{y_{t+1}}{y_t}$$

Other measure of growth: the time it takes per capita income to double.

$$t = \frac{\ln 2}{g}$$

Fig 1: Real per capita GDP in the US, 1870-1994.

Long run growth among countries after their industrial revolution:

GDP per capita in 1985$

	1820	1989	ratio
France	1052	13837	13.2
Germany	937	13989	14.9
UK	1405	13468	9.6
US	1048	18317	17.5
Japan	588	15101	25.7
Italy	960	12955	13.5

Evolution of labor productivity (in dollars 1985) Labor productivity is GNP per worker Y_t/N_t.

	1890	1913	1950	1973	1987	1987/1890
France	1.52	2.26	4.58	14	21.63	14.2
Germany	1.52	2.23	3.40	12.83	18.35	12.1
UK	2.86	3.63	6.49	13.36	18.46	6.5
US	2.82	4.68	11.39	19.92	23.04	8.2
Japan	0.58	0.86	1.69	9.12	14.04	24.2
Italy	0.99	1.72	3.52	12.82	18.25	18.4

But, in the very long run, steady growth is unusual (cf. some historical examples in Fig 2-3).

Fact 2 : GDP per capita and rates of economic growth vary substantially across countries

Inequalities in the distribution of the world income (Fig 4-5).

Inequalities in growth rate. Are poor countries catching up rich one ? (Fig 6)

Others facts

Fact 3 : A steady growth rate for the capital stock per capita.

Fact 4 : the real rate of return to capital r shows no trend upward or downward.

Fact 5: A steady ratio of the capital over real GDP K/Y.

	1890	1913	1950	1973	1987
France		1.64	1.68	1.75	2.41
Germany	2.29	2.25	2.07	2.39	2.99
UK	0.95	1.03	1.10	1.73	2.02
US	2.09	2.91	2.26	2.07	2.30
Japan	0.91	1.01	1.80	1.73	2.77

Fact 6: the shares of income devoted to capital rK/Y and labor wN/Y show no trend upward or downward.

1.2 The basic model : Solow (1956)

Solow's model is a simple model that fits Kaldor's stylized facts.

Basic assumptions: One aggregate good in the economy which can be used as capital or consumption good.

The economy is perfectly competitive on all markets : labor market, good (capital) market.

1.2.1 The aggregate production technology and the firm behavior

An aggregate production technology with the following properties:

$$Y = F(K, L)$$

F is linear homogenous, with substitutable factors (F is continuously differentiable), increasing in both arguments, concave.

Cobb-Douglas: $F(K, L) = AK^{\alpha}L^{1-\alpha}$.

CES: $F(K, L) = A(K^{1-1/\sigma} + bL^{1-1/\sigma})^{\frac{\sigma}{\sigma-1}}$

K results from past investments:

$$K_{t+1} = I_t + (1-\delta)K_t$$

with $0 < \delta < 1$ the depreciation rate of capital.

Considering a firm in period 0, its objective is intertemporal profit maximization. If r_t is the real interest rate (between $t-1$ and t), and w_t the real wage, the intertemporal gain of the firm (in $t=-1$) is:

$$\sum_{t=0}^{+\infty} \frac{F(K_t, L_t) - w_t L_t - I_t}{\rho_t}$$

with

$$\rho_t = \prod_{i=0}^{t} (1 + r_i)$$

It is possible to write this gain:

$$\sum_{t=0}^{+\infty} \frac{F(K_t, L_t) - w_t L_t - (r_t + \delta) K_t}{\rho_t} + K_0$$

At each date t, firm's objective is static: to maximize its current profit

$$F(K_t, L_t) - w_t L_t - (r_t + \delta) K_t$$

Firm's behavior:

$$\max_{(K_t,L_t)} F(K_t, L_t) - w_t L_t - (r_t + \delta) K_t$$

leads to first order conditions

$$\begin{aligned} F'_{K_t}(K_t, L_t) &= (r_t + \delta) \\ F'_{L_t}(K_t, L_t) &= w_t \end{aligned}$$

or the Factor Price Frontier.

1.2.2 The Solow's residual

Solow (1957) introduces growth accounting.

Assume an aggregate production technology:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

with the approximation:

$$\frac{\Delta X_t}{X_t} = \ln X_{t+1} - \ln X_t$$

for the growth rate, Solow's residual is defined by:

$$\frac{\Delta Y_t}{Y_t} - \alpha\frac{\Delta K_t}{K_t} - (1-\alpha)\frac{\Delta L_t}{L_t}$$

α is the share of capital revenues in the total output (assuming perfect competition), $1-\alpha$ the share of labor revenues.

L is the quantity of hours, K the capital stock.

Output growth cannot be explained only by the growth of production factors.

The residual: a "free" hidden factor, a technical progress.

The hidden factor explain the main part of output growth.

labor
+ duration
=working time (1)
Physical capital (2)
GNP growth rate (3)
Solow's residual=(3) - 0.7 (1) - 0.3 (2)

	France			Germany	
13-50	50-73	73-87	13-50	50-73	73-87
0.04	0.43	0.02	0.55	1.08	0.0
-0.8	-0.36	-0.98	-0.3	-1.08	-0.77
-0.76	0.06	-0.96	0.24	-0.02	-0.77
1.21	5.12	4.49	1.06	6.60	3.45
1.15	5.04	2.16	1.28	5.92	1.8
1.31	3.46	1.48	0.79	3.95	1.30

	US			Japan	
13-50	50-73	73-87	13-50	50-73	73-87
1.29	1.51	1.93	0.89	1.7	0.84
-0.9	-0.36	-0.47	-0.48	-0.15	-0.25
0.39	1.15	1.46	0.41	1.55	0.59
2.09	3.24	3.28	3.85	9.08	7.57
2.79	3.65	2.51	2.24	9.27	3.73
1.89	1.87	0.5	0.79	5.46	1.04

Remark: This is the naïve Solow's residual. Some corrections can be introduced (imperfect competition, labor heterogeneity and human capital, embodied technical progress ...)

1.2.3 The production function with technical progress

Basic idea: the production technology depends on time.

$$\begin{aligned} Y_t &= F(K_t, A_t N_t) \\ N_t &= N_0 (1+n)^t \\ A_t &= A_0 (1+a)^t \end{aligned}$$

technical progress neutral in Harrod sense (resp Hick, Solow).

Only Harrod's formulation allows a balanced growth path in the Solow's model when the production technology is general. When F is Cobb-Douglas, all forms are equivalent.

1.2.4 Individual behaviors and accumulation

Labor quantity increases at a constant rate n.

Households consume a constant share of total income:

$$\begin{aligned} C_t &= (1-s)Y_t \\ S_t &= sY_t \end{aligned}$$

Capital accumulation:

$$K_{t+1} = K_t(1-\delta) + I_t$$

Equilibrium of the capital (good) market:

$$S_t = I_t$$

Notation: $k_t = K_t/(A_t N_t)$.

$$\begin{aligned} k_{t+1} &= \frac{1}{(1+n)(1+a)} \left[s f(k_t) + (1-\delta) k_t \right] \\ \frac{k_{t+1} - k_t}{k_t} &\approx \frac{s f(k_t)}{k_t} - \delta - n - a \end{aligned}$$

1.2.5 The dynamics

The dynamics is monotonic.

The Cobb-Douglas case.

Two stationary states, one stable k^*, one 0 unstable.

$$k^* = \left(\frac{s}{\delta + n + a + (an)}\right)^{\frac{1}{1-\alpha}}$$

1.2.6 Long run properties of the economy

$$\begin{aligned} F(K_t, A_t N_t) &= F_L(k_t, 1) A_t N_t + F_K(k_t, 1) K_t \\ &= w_t N_t + (r_t + \delta) K_t \end{aligned}$$

Comparison with Kaldor's stylized facts.

Impact of the different variables: s, n, a.

1.2.7 The optimal long run path

The problem of Solow-Phelps: the saving rate which maximizes consumption per head in the long-run.

$$c_t = \frac{C_t}{N_t} = \frac{(1-s)Y_t}{N_t} = A_t(1-s)f(k^*(s))$$

with $k^*(s)$ implicitly defined by

$$sf(k^*(s)) = (\delta + n + a)k^*(s)$$

The golden rule:

$$r = f'(k^*(s)) - \delta = n + a$$

$r > n + a$: under-accumulation.

$r < n + a$: over-accumulation.

Over-accumulation is inefficient.

1.3 Analysis of convergence

1.3.1 Convergence in Solow's model

We consider the output per capita: $y_t = Y_t/N_t = A_t(k_t)^\alpha$.

We have

$$\begin{aligned} y_t/y_t^* &= \left(\frac{k_t}{k^*}\right)^\alpha \\ \ln\left(y_t/y_t^*\right) &= \alpha \ln\left(\frac{k_t}{k^*}\right) \end{aligned}$$

The dynamics of k_{t+1} can be written:

$$\ln\left(\frac{k_{t+1}}{k_t}\right) = s\left(k_t^{\alpha-1} - k^{*\alpha-1}\right)$$

and

$k_t^{\alpha-1} - k^{*\alpha-1} = \exp\left[(\alpha-1)\ln k_t\right] - \exp\left[(\alpha-1)\ln k^*\right] \approx$

$k^{*\alpha-1}(\alpha-1)(\ln k_t - \ln k^*) = \frac{a+n+\delta}{s}\frac{\alpha-1}{\alpha}(\ln y_t - \ln y_t^*)$

Finally:

$$\begin{aligned}
\ln\left(\frac{y_{t+1}}{y_t}\right) &= a + \alpha \ln\left(\frac{k_{t+1}}{k_t}\right) \\
\ln\left(\frac{y_{t+1}}{y_t}\right) &= a - (1-\alpha)(a+n+\delta)(\ln y_t - \ln y_t^*)
\end{aligned}$$

with $\ln y_t^* = at + \ln A_0 + \alpha \ln k^*$

Consequence: empirical test of convergence. Countries starting from a low level of income per capita should grow at a higher rate ($\beta-$convergence).

1.3.2 Empirical tests of convergence

Estimation of the equation:

$$\ln\left(\frac{y_T^i}{y_0^i}\right) = a - \beta \ln y_0^i + \varepsilon^i$$

where i is the index of the country. 0 is the reference date.

From Solow's model, this estimation should be good if the different countries of the sample have the same economic parameters $(s, \alpha, a, \delta, n)$, that is: same technologies, same technical progress, same saving rate.... The constant a may depend on the reference date 0.

Results: good when countries are close, bad when the sample of countries is large.

cf; Baumol, De Long, Barro and Sala-i-Martin.

When countries are heterogenous, a test of "conditional convergence" is introduces:

$$\ln\left(\frac{y_T^i}{y_0^i}\right) = a - \beta \ln y^i + \sum_j \gamma_j x_j^i + \varepsilon_t^i$$

x_j^i are different economic variables (ex: schooling, fertility rates, political instability, public expenses etc...), γ_j are estimated coefficients.

Other test of convergence: $\sigma-$convergence. There is $\sigma-$convergence when the variable $\text{Var}\left(\ln y_t^i\right)$ is decreasing with t.

1.3.3 Results:

Data: 3 periods are studied. 1965-75 (72 countries), 75-85 (86 countries), 85-95 (83 countries).

Explanatory variables:

- Initial Per Capita GDP: the variable is $\ln(GDP)$ in 65, 75 and 85. Estimated coeff = -0.0248

- Educational Attainment: var = average years of male secondary and higher schooling (upper-level schooling) observed respectively in 65, 75, 85. (An one year increase in male upper-level schooling raises the growth rate by 0.0036). Estimated coeff = 0.0036.

- Life expectancy taken in 60, 70, 80. var = 1/(Life expectancy at age 1) = "average probability to died at each period". Estimated coeff = -5.04.

- Fertility Rate: ln(total lifetime live births for a woman over her expected life. Taken in 60, 70, 80. Estimated coeff = -0.0118.

- Government Consumption Ratio: (real government consumption)/(real GDP). In real government consumption, defense spending and education expenditures are subtracted. The high value is consistent with the fact that the variable is a ratio. Estimated coeff = -0.062.

- Rule of Law: an index between 0 and 1, built using subjective measure. 1 = the most favorable environment for maintenance of the rule of law. These data begin in 82. Estimated coeff = 0.0185.

- Democracy: an index between 0 and 1, built using subjective measure of electoral rights. 0 = a complete totalitarian system, 1 = a full representative democracy. These data begin in 72. A non-linear relation: democratization enhances growth for countries that are not democratic, but retards growth for countries that have already a substantial amount of democracy. Estimated coeff = 0.079 and -0.074 (squared).

- International Openness = (Exports + imports)/GDP. Estimated coeff = 0.0054.

- Terms of Trade = Growth rate of the terms of trade (export prices relative to import prices) over each ten year period. Estimated coeff = 0.130

- Investment ratio = (gross domestic investment (private and public))/(real GDP) as averages for each of the ten-year periods. Estimated coeff = 0.083

- Inflation Rate = the average rate of inflation over each ten-year periods. Estimated coeff = -0.0119.