Life Expectancy, Growth and PAYG Pension Systems

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Very First Draft

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Abstract : The increasing number of people aged 65 and more has a significant impact on public spending and notably on pay-as-you-go pension systems. The aim of this article is to explain the inverted U-shaped relationship between the growth rate of the economy and life expectancy with regards to the generosity of the pension system. We propose two main explanations for this fact. The first one concerns the effect of life expectancy on saving and on the tax rate necessary to finance pensions for retired people. Indeed, when life expectancy increases there is a greater incentive for people to save to finance their additional consumption. But, as people live longer, people receive pensions during a longer period of their life, so the tax rate increases. This increase has a negative impact on saving and so on the growth rate. We also study the impact of the growth rate of the population and of the minimum legal age to retire on this growth rate.

The second explanation concerns the impact of life expectancy on growth but using a human capital approach and a pension system. We show that as long as life expectancy is not too high the increase of life expectancy increases the time devoted to education and the growth rate as public spending for education are not so affected by the increase in the tax rate of the pension system. But once life expectancy is high, spending for education decrease so that we observe simultaneously an increase of the time devoted to education and a reduction of the growth rate.

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1 Introduction

Life expectancy at birth increases from 68.9 in 1950, to 73.3 in 1975, and reaches 77.5 in 2000[2] for the United States. Furthermore total fertility rates for the same period are respectively : 3.45, 1.79 and 1.93[3]. These two combined effects imply an increase of the retiree to worker dependency ratio. This ratio is 24.5 in 2000, and is expected to be 42 in 2030[4]. This increase was amplified because people retire earlier. Indeed, males in United States retired at 64.2 on average in 1980, but at 63.6 in 2000.
The increase of the life expectancy can have effects on the growth rate. Empirically, some authors have found an inverted U-shaped relationship between the growth rate of GDP per capita and life expectancy. Zhang and alii [2003] sum up quantitative results of Barro and Wolf [1989] about this relationship (See table 1).

Life expectancy at birth	<60	60-64	65-69	≥70
Number of countries	41	8	14	12
Private Investment/GDP (1970-1985(%))	14	20	23	22
Average Growth rates (1960-1985(%))	1.88	3.18	3.36	2.50

We see that in a first time, an increase of life expectancy has a positive effect on the growth rate, but a negative one when life expectancy becomes high. This table exhibits the impact of an aging population on the growth rate of an economy. Tabata [2005] proposes a relationship between the old age dependency ratio and the growth rate. He finds the same qualitative results as those of Zhang and ali. [2003]. When this dependency ratio reaches 15%, all increase of life expectancy reduces the growth rate.

Some theoretical explanations have already been given to explain this relationship. First, arguments that explain the positive relationship are : an increase of saving since people save more to finance their additional consumption (Zhang and alii [2003] , Cipriani [1999]) ; an increase of investment for education and a decrease of the depreciation rate of the human capital (De la Croix and Licandro [1999]). Secondly, arguments that explain the negative part of the relationship are : a decrease of unintentional bequests (Cipriani [1999], Zhang and alii [2003]) ; an increase of costs for health cares (Tabata [2005]) ; an increase of the number of people who do not have a great human capital (De la Croix

[2] From Nyce and Schieber [2005], pp.15

[3] Ibid, pp.18

[4] Ibid, pp.63

and Licandro [1999]) ; a decrease of the investment for education since the median voter is older (Zhang and alii [2003]).

What is interesting is that at any times the problem related to retirement has not been studied. Nevertheless a lot of countries worry about the economic impact of their pension system, and notably in European countries since their pension system is essentially an unfunded system (a Pay-As-You-Go system). We argue that two kinds of arguments can be given. First we focus on the interaction between the PAYG pension system and the accumulation of physical capital. The main channel concerns the effects of the increasing life expectancy on saving and on the tax rate. While the second channel focus on the fact that increasing expenditures for pensions decreases disposable resources for education, and therefore, this can reduce the growth rate of the human capital.

In section 2, we focus on the accumulation of physical capital. The aim of this part is to explain why the improvement of life expectancy can have two kinds of basic effects on the growth rate with a pay-as-you-go pension system. The first one is the increasing incentive for people to save, because they have to finance an additional consumption. The second one is the increasing tax rate necessary to finance pensions for retired people. We show that the "save effect" is first the most important, so that the growth rate is an increasing function of life expectancy. But, once life expectancy reaches a threshold, the "tax effect" is greater and growth rate depends negatively on life expectancy. We also show that the growth rate of population[5] have a first negative impact on growth because of a dilution effect on capital, but that it has a positive impact on the growth rate if life expectancy is sufficiently high since it reduces the tax rate per capita. Finally, we exhibit the impact of the increase of the minimum retirement age. We show that this has three effects. The first is a dilution effect since there is less capital per worker. The second is a "constraint effect". Since agents work during a longer time in their second part of life they will reduce their saving. The third effect is that this increasing working period has a negative impact on the tax rate, that can increase saving and so the physical capital accumulation.

In section 3, our approach is different since we try to exhibit the impact of the increasing spending for the pension system on alternative spending like these for education. We use a human capital model. We show that the increasing life expectancy have a positive

[5] It represents the evolution of the fertility rate.

impact on time spent for education that has a positive impact on the growth rate of the human capital. But this increasing time has too a negative impact on time spent at work that reduce tax revenues and so public education spending. But we show that the main channel concerns the increase of spending for the pension system, that reduces disposable resources for education. We also give the impact of an increasing growth rate of population on the growth rate of the human capital. We show that as long as life expectancy is low, it has a negative impact since it reduces education spending per child. But once life expectancy is high it permits to reduce public spending for the pension system and life expectancy has a net positive effect on the growth rate.

In section 2, we describe our physical capital approach. In section 3, we proceed the same way but with a human capital approach. In section 4, we give some concluding remarks.

2 The physical capital approach

We first present some facts first that describes he evolution of the parameters of the PAYG pension system in a context of aging societies. Given these facts, we construct a model that try to reproduce it. We also study the main properties of this dynamic.

2.1 The generosity of public pension systems

What is interesting is to know how evolved the generosity of public pension systems during the aging process, and notably to know if it is the tax rate or the replacement rate that adjusts.

First, the gross replacement rate of earnings at least remains constant for all developed countries (Nyce and Schieber [2005], pp.236) :

Countries	1975	1995
Belgium	0.6	0.6
France	0.50	0.65
Germany	0.55	0.54
Italy	0.62	0.80
Japan	0.48	0.44
United Kingdom	0.21	0.39
United States	0.35	0.42

It means that public pension systems have become more generous since 1975 although the number of old people increased quickly during this period. It implies logically that the tax rate has increased during this period, and it is what we observed (Payroll Tax rates for Various Years under old Age Pension Programs, Ibid, pp.238) :

Countries	1967	1995
Belgium	12.5	16.4
France	8.5	19.8
Germany	14	18.6
Italy	15.8	29.6
Japan	5.5	16.5
United Kingdom	6.5	13.9
United States	7.1	12.4

So, when the old age dependency ratio increased the public pension systems became more generous. It could explain why life expectancy can have an adverse effect on growth since government have to finance this generosity by taxes that reduce the disposable income and so saving. We now study a model that takes into account these characteristics. Intuitively we can foresee that as long as the old age dependency ratio is small every increase in life expectancy increase strongly the saving but has only a small effect on the tax rate. But when this old age dependency ratio is great, that is when life expectancy is high, if the system remains generous, the effect on the tax rate becomes predominant, and every increase of life expectancy has a negative impact on the growth rate.

2.2 The model

The economy is composed in a period t of two generations, a young and a old generation. Each young of generation t gives birth to $1+n$ children[6], work during his first part of life and a part of the second one. All agents have the same preferences, and there is no uncertainty in this economy. Utility functions are intertemporally separable. Utility depends on consumption for the two periods, but the utility of the second period depends too on leisure that is to say time that is not spent at work. To represent these preferences, the model of d'Autume [2003] is used.

This model will also be interesting because we can distinguish between the effects of life expectancy and the effects of the fertility rate. Here we assume that people live only a fraction T of their second period of life[7]. It means that $1 > T > 0$. The utility of a child born in period t is :

$$U_t \equiv u(c_t) + \beta T u(d_{t+1}/T; (T - z_t)/T) \tag{1}$$

β is the psychologic actualisation factor, whereas c_t and d_{t+1} denote respectively the consumption during the first and the second period. $(T - z_t)$ is the leisure time. Since the agent do not live during all the second period what matters for him is the flow of consumption and of leisure, that is why it is divided by T.

Because young people offer inelastically their workforce they obtain a wage w_t. But a fraction τ_t from this wage is levied to finance the pension system. Agents save a level S_t for their second period consumption. During this last period the agent receive his saving augmented by interests R_{t+1}, a net wage $((1-\tau_{t+1})w_{t+1})$ during the time that he works, and a pension p_{t+1} during his retirement period $(T - z_t)$. We assume that this pension corresponds to a fraction of the current wage $(\lambda_{t+1} w_{t+1})$. This simplifies the resolution of the model. Consequently, his budget constraints are :

$$\begin{aligned} c_t &= w_t(1-\tau_t) - S_t \\ d_{t+1} &= S_t R_{t+1} + z_t(1-\tau_{t+1})w_{t+1} + (T - z_t)\lambda_{t+1} w_{t+1} \end{aligned}$$

Agents maximize their utility under the budget constraints. We assume that agents have a logarithmic utility function during the two periods that is $u(x) = \ln(x)$, but his

[6]Here, the fertility rate is exogenous. Endogenize this rate would induce to make assumptions about the way people take into account all consequences of the fertility rate. See Wigniolle and Loupias [2004] for more details.

[7]See d'Autume [2003] for more details.

second period utility function is composed of the consumption and of the leisure flows. We assume that it has the following form : $\ln\left(\left(\frac{d_{t+1}}{T}\right)^m\left(\frac{T-z_t}{T}\right)^{1-m}\right)$.

The two control variables are S and z, and one obtains :

$$S_t = w_t(1-\tau_t)\frac{\beta T}{1+\beta T} - \frac{w_{t+1}}{R_{t+1}}\left[\frac{(1-\tau_{t+1})(1+\beta T)-\lambda_{t+1}}{\beta(1+\beta T)}\right]$$

$$z_t = T\frac{1+m\beta T}{1+\beta T} - (R_{t+1}w_t(1-\tau_t) + T\lambda_{t+1}w_{t+1})\frac{(1-m)\beta T}{w_{t+1}(1-\tau_{t+1}-\lambda_{t+1})(1+\beta T)}$$

Lemma 1 : A sufficient condition for agents choose the corner solution ($S_t > 0$ and $z_t = 0$) is that the replacement rate λ_{t+1} is sufficiently large, and particulary if $\lambda_{t+1} > 1 - \tau_{t+1}$.[8]

This condition implies that the agent choose to not work during his second part of life if the marginal return on working $(1-\tau_{t+1}-\lambda_{t+1})$ is negative. If we have an interior solution, so saving is an increasing function of w_t. This increase permits the agent to work less during his second period of life (z_t decreases). An increase of w_{t+1} imply that the agent prefers to consume more during his first period of life (S_t decreases) and to work more during the second one. What is particulary interesting concerns the impact of the replacement rate λ_{t+1}. An increase of it imply that the agent prefers more saving and working less during his second part of life. Indeed, an increase of the replacement rate implies that the return on working decreases, so the agent increases his leisure time. But this increase of the pension is not sufficient to obtain the desired level of consumption that is why saving increases.

We have now to specify the equilibrium of the pension system :

$$N_t w_t \tau_t + N_{t-1} w_t \tau_t z_{t-1} = N_{t-1}\lambda_t w_t (T - z_{t-1})$$

We assume that the level of pensions depends on the level of wages in the same period[9] and that taxes that come from old workers are used to finance pensions of these people during the period considered. z_{t-1} is the time spent at work by old people during period t. In accordance with empirical findings, we assume for the rest of the paper that the

[8] This result comes from the study of the first order conditions.

[9] See d'Autume [2003].

replacement rate λ is constant and that the tax rate τ adjusts so that the the social security constraint is at equilibrium at every periods. Finally, we have :

$$\tau_t = \frac{T - z_{t-1}}{1 + n + z_{t-1}} \lambda$$

The first term in the right hand side of the equation is the dependency ratio, and the second term is the replacement rate. The more generous the system is, or the larger the dependency ratio is, the greater the tax rate has to be.

To obtain a general equilibrium analysis we have to specify the behavior of firms. In every period there is a large number of firms. Each of them uses labor and capital to produce a unique final good. This good is the numeraire of the economy. There are constant returns to scale for private factors of production :

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

A_t mesures the level of technology of the economy, and more specifically[10] :

$$A_t = a \left(\frac{K_t}{L_t} \right)^{1-\alpha} = a k_t^{1-\alpha}$$

with a an exogenous parameter strictly positive, and k which mesures the capital per worker. It means that the knowledge of the economy depends on the level of capital per active worker. The more an economy can bring capital per worker, the more they can learn or the better they can do their work. It is a modified Romer [1986] technological progress. This form enables us to obtain an endogenous growth, that is a constant growth rate of capital per capita. We assume that firms do not take into account the impact of their decisions on the technological level of the economy so wages and interest rate are :

$$\begin{aligned} w_t &= a(1-\alpha)k_t \\ R_t &= a\alpha \end{aligned}$$

We have a well property for this two variables since wages will evolve according to the accumulation of capital worker, whereas the interest rate will be constant.

[10] This form is the same as this used by Frankel [1962].

2.3 The dynamic and its properties

Let us now specify more assumptions that better represent empirical findings. We will assume that there exists a minimum legal age to retire that we note $\underline{z}$. Moreover we assume that agents choose the corner solution : $S_t > 0$ and $z_t = \underline{z}$.

To obtain the dynamic of the economy we note that the saving in period t is used by firms to finance their capital of the next period so that at equilibrium we have : $N_t S_t = K_{t+1}$, or equivalently :

$$S_t = (1 + n + \underline{z})k_{t+1}$$

where k mesures capital per worker.

Using the saving equation and the values of the tax rate, wages and interest rates one obtains :

$$\frac{k_{t+1}}{k_t} = g(T, n, \underline{z}) = \frac{a\beta T m(1-\alpha)\left(1 - \frac{T-\underline{z}}{1+n+\underline{z}}\lambda\right)}{(1 + m\beta T)(1 + n + \underline{z}) + \frac{1-\alpha}{\alpha}\left(\underline{z}\frac{1+n+\underline{z}-\lambda(1+n+T)}{1+n+\underline{z}} + \lambda T\right)}$$

If $(g(T, n,\underline{z}) - 1)$ denotes the growth rate of capital per worker of the economy[11]. We have an endogenous growth if $g(T, n,\underline{z}) > 1$. This means that we have to make an assumption on a so that this condition is checked. But as the length of life can be very small this would impose that the parameter a has to be great in this case. That is why we make only the following assumption which ensures that there exists a threshold $T_{\inf}$, beyond which $g(T, n, \underline{z}) > 1$:

$$a > \frac{(1 + m\beta)(1 + n + \underline{z}) + \frac{1-\alpha}{\alpha}\left(\underline{z}\frac{1+n+\underline{z}-\lambda(2+n)}{1+n+\underline{z}} + \lambda\right)}{\beta m(1-\alpha)\left(1 - \frac{1-\underline{z}}{1+n+\underline{z}}\lambda\right)}$$

We now have to study the properties of the growth rate of the economy when the life expectancy and the growth rate of the population vary. The aim of this part is to

[11] We show very easily that the growth rate of GDP per capita is the same as the growth rate of GDP per worker. Indeed, we have : $\tilde{y}_t = \frac{Y_t}{N_t + TN_{t-1}} = \frac{Y_t}{L_t}\frac{1+n+z}{1+n+T} = y_t\frac{1+n+\underline{z}}{1+n+T}$, with y_t GDP per worker. We know thanks to the production function that $y_t = ak_t$ once the externality is taken into account. So at the steady state we have : $g = \frac{\tilde{y}_{t+1}}{\tilde{y}_t} = \frac{y_{t+1}}{y_t} = \frac{k_{t+1}}{k_t}$.

show the impact of the pension system on the growth rate when life expectancy increases. Furthermore, we show the importance of fertility when population is ageing.

Until recently the minimum legal age to retire was 60. If we consider that one period represents 40 years, so it implies that $\underline{z}= 0$, that is to say that agents retire at the beginning of their second period of life. We will in proposition 3 study the impact of an increase of $\underline{z}$.

Let us first study the impact of the increase of life expectancy.

Proposition 1 : If $\lambda > \frac{1+n}{2+m\beta}$, for $T \in (0, T_{\min})$[12] $g(T,n)$[13] is an increasing function of T. But for $T \in (T_{\min}, 1)$ it is a decreasing function of T.

Proof : We have to calculate the derivative of $g(T,n)$ relatively to T. One obtains an equation of the form : $\frac{-bT^2-cT+d}{()^2} = \frac{P(T)}{()^2}$, with b, c and d $\in \mathbb{R}^+$. We show easily that $P(0) > 0$ for the parameters of the model. For $g(T,n)$ is decreasing with respect to T, a necessary and sufficient condition is that $P(1) < 0$. This condition is given in the proposition.

There are two effects related to the length of life in the model. The first one is called the "saving effect" since the increasing length of life implies that agents have to save more to finance their additional consumption. But a longer length of life implies that workers have to finance pensions during a longer period. We call this last effect the "tax effect". While the length of life is not too high the first effect dominates because the dependency ratio is still not too high. But when the length of life is above the threshold the size of the retired population is sufficiently important so that every rise in length of life will strongly affect the tax rate and finally the net impact will be negative.

There is another way to read this proposition. In a first part of development countries do not have any social security program so that only "saving effect" is relevant. Then, the wealth of these countries permits them to develop this kind of programs without worrying consequences since the dependency ratio is not too high. Finally, the generosity of the system can not be reduced dramatically although life expectancy rise quickly. Consequently the "tax effect" becomes more important.

Our aim is not to explain the appearance of the social security programs but it helps to explain the analytical proposition.

[12] The condition for a ensures that $T_{\min} > T_{\inf}$.

[13] Until proposition 3 we consider that $\underline{z}=0$, that is why we note $g(T,n,0) \equiv g(T,n)$.

Another important point of this article concerns the impact of the growth rate of the population n on the growth rate of the economy.

Proposition 2 : For n sufficiently small and for λ sufficiently large, while $T \in (0, T_{tild})$ we have $\frac{\partial g(T,n)}{\partial n} < 0$, but $\frac{\partial g(T,n)}{\partial n} > 0$ for $T \in (T_{tild}, 1)$.

Proof : The derivative of $g(T, n)$ with respect to n gives an equation of the form : $\frac{eT^2+fT-g}{()^2} = \frac{P(T)}{()^2}$. The condition on n ensures that $P(1) > 0$, knowing that $P(0) < 0$.

There are two effects of the growth rate of the population on the dynamic of the economy. The first one is the "dilution effect", and the second one is the "tax effect". The first implies that an increase of the fertility rate reduces the capital stock per capita and so reduce the growth rate of this capital per capita. But the second one implies that this increase of the number of children reduces the tax per capita that has to be paid to finance the pension system and so increases the level of saving. The second effect becomes the most important once the dependency ratio is large enough.

This proposition can explain why countries have a political incentive first to reduce their fertility rate when their level of development is not high because it has a negative impact on the growth rate. But, once these countries adopt a social security program and have a long length of life, they worry about their fertility rate because it could reduce the tax rate per capita.

Let us now study the impact of an increase of the minimum legal age to retire ($\underline{z}$), and particulary the impact of that increase relative to the point where $\underline{z}= 0$. Our aim is to show that recent reforms of pension systems in Europe are relevant because life expectancy is high, but that adjustments by $\underline{z}$ are irrelevant in a context where life expectancy is low.

Proposition 3 :For $\underline{z}\rightarrow 0$, for $T \in (0, \hat{T})$, $\frac{\partial g(T,n,\underline{z})}{\partial \underline{z}} < 0$ but for $T \in (\hat{T}, 1)$, $\frac{\partial g(T,n,\underline{z})}{\partial \underline{z}} > 0$.

Proof : The derivative of $g(T, n,\underline{z})$ with respect to $\underline{z}$ when $\underline{z}$ is 0 gives an expression of the form : $\frac{hT^2+iT-j}{()^2} = \frac{P(T)}{()^2}$. We show very easily that $P(0) < 0$ and that $P(1) > 0$.

$\underline{z}$ has three effects on the growth rate. The first one is a "dilution effect" as for the impact of n. An increase of $\underline{z}$ implies that there is less capital per worker and so on that the growth rate of the capital per worker decreases. The second effect is a "constraint effect". The more the agent has to work during his second part of life, the more will be his incomes during this period and the less he has to save to finance his second period consumption. This decrease of saving has a negative impact on the growth rate of capital per worker. The third effect is a "tax effect". When agents work longer, the tax rate on work decreases and therefore has a positive impact on saving.

From this proposition we can see that countries in which life expectancy is high can increase their growth rate by increasing the minimum legal age to retire. This can too explain why this solution has been used so late. It is because the two first negative effects are the most important when life expectancy is low.

3 The human capital approach

The aim of this section is to show that an increase of life expectancy can increase the tax rate used to finance a Pay-As-You-Go pension system and so on decrease spending for education. This last effect can reduce the growth rate of the economy. We first present some empirical evidences of the impact of aging population on education spending and related literature. Then we present our model and prove the existence of an inverted U-shaped relationship between life expectancy and growth.

3.1 Empirical evidence and related literature

In this section we present some articles that argue that an aging population can have an effect on education spending and so on the growth rate.

Related literature

Some theoretical articles have already emphasized the link between the life expectancy, the human capital and the growth rate of an economy.
Zhang and al. [2003] explain the inverted U-shaped relationship between life expectancy and growth by three factors. The first is the increase of the saving rate. The second and the third are the decrease of accidental bequests and the decrease of the tax rate for public education. This last effect is an important component of the model and it is determined endogenously by a median voter procedure. Indeed when longevity is higher people need a higher saving rate to finance their old-age consumption since their chance of surviving is improved.
In a political economy model, Kemnitz [2000] prove that when life expectancy increases

the preferred tax rate for education increases in the presence of a PAYG pension system since it raises tomorrow's workers wages and so futur pensions benefits. Therefore in this model life expectancy has a positive effect on the growth rate.

Non-public choice models are those based on the model of de la Croix and Licandro [1999]. This model permits too to obtain the inverted U-shaped relationship between life expectancy and growth. Human capital is the only engine of growth. Human capital per capita depends on the time dedicated to education and on the average human capital. People choose the time of this period and the age at which they decide to retire. Several generations coexist at the same time and each of them is endowed with a level of human capital[14]. The increase of life expectancy has three effects : (1) a decrease of the depreciation rate of the human capital ; (2) an increasing time dedicated to education ; and (3) an increasing part of people with low human capital. Therefore, the decreasing part of the inverted U-shaped relationship is explained by a vintage human capital argument.

Echevarria and Iza [2006] use another version of this model (this of Boucekkine and al. [2002]) and show that with a PAYG pension system the vintage human capital argument is not the more relevant to explain the decreasing part of the relationship between the life expectancy and the growth rate. They show that when life expectancy increases the main effect is that people retire earlier and so the GDP per capita grows less quickly.

Our model will use a human capital structure. But the main argument is not the same as the two previous articles. We use the fact that the replacement rate used in the PAYG pension system is a parameter contrary to Echevevarria and Iza [2006]. Furthermore, the tax rate is assumed to be given so our model is not a political economy model. Our aim is to show that the increase of life expectancy has a positive impact on time dedicated to education, but that this effect can be compensated by a decrease of education spending since more public founds have to be used to finance the pension system. We will combine some arguments of the previous articles and show that we also can obtain the inverted U-shaped relationship between the life expectancy and the growth rate per capita, and so to exhibit a new channel for the explanation of this relationship.

But firstly, we have to give some arguments that justify the negative relationship between life expectancy and education spending.

Life expectancy and education spending

[14]It is a model of overlapping generations with continuous time.

Poterba [1997] found that the more is the share of elderly residents, the less is per-child education spending by using panel data for the states of the united states over the 1960-1990 period. More precisely he finds an elasticity of education spending relatively to the share of elderly residents of -0.25. Harris and al. [2001] also find this kind of correlation at the state level but with a lower elasticity. But they moderate this argument since it is not observed at the district level.
Grob and Wolter [2005] try to exhibit the same kind of relationship for the Swiss Cantons for the period 1990-2002. They found that the share of the elderly has a significant negative influence on the willingness to spend on public education.

Our argument is not to say that the increasing share of the elderly makes that a new political gerontocraty emerges, but more simply that the increasing share of elderly people implies new costs for the society. These costs can be used for medicare (Tabata [2005]), or to finance the pension system. It is this last effect that we highlight. Indeed, an increasing life expectancy implies first an increasing time dedicated to education that has a positive impact on the human capital accumulation. But in the same time, pensions have to be given for a longer period, and so reduces public expenditures for education. This last effect has a negative impact on the human capital accumulation. We will show that the first effect dominates when life expectancy is not too high while the second becomes relevant when life expectancy is high.

3.2 The model with human capital

The behavior of consumers, firms and government will first be presented. We then study the dynamic of the model and specify the growth rate of the economy and its properties.

Consumers

Let us first assume that agents have the same preferences as in the previous part (equation (1)). Furthermore we assume that agents can choose time they spend for education (e_t). In this model there is no direct disutility of education[15]. This education has a positive impact on human capital of the agent (h_t) so that :

$$h_t = h_{t-1}^{1-\xi} \left(\frac{G_{t-1}}{N_t} \right)^{\xi} e_t^{\xi} \tag{2}$$

[15] The length of each period is normalized to 1.

where h_{t-1} is the average human capital of the previous generation and G_{t-1} represents public spending for education from the previous period. We also assume that $0 < \xi < 1$. We assume that wages are reduced by a tax that is used by the government to finance the pension system and education spending. The agent takes only into account the direct impact of his education choice on the human capital accumulation.

Finally the budget constraints of agents are :

$$c_t = w_t(1 - \tau_t)(1 - e_t)h_t - S_t \tag{3}$$

$$d_{t+1} = RS_t + h_t w_t((1 - \tau_{t+1})z_t + \lambda(T - z_t)) \tag{4}$$

with w_t the wage per unit of efficient labor. For simplicity, we assume that agents adopt a corner solution for their labor supply during their second life period.

Assumption 1 : τ and λ are sufficiently high so that agents choose the corner solution $z_t = \underline{z}$.

Assumption 2 : For the clarity of the text we assume that $\underline{z} = 0$.

As saving do not play an important role in this model we do not have to determine it. What matters is the education behavior. As education has no direct impact on utility, agents determine time spending for education in maximizing his actualized budget constraint, and we obtain :

$$e_t = \frac{\xi}{1+\xi}\left[1 + \frac{\lambda T}{R(1-\tau_t)}\right] \tag{5}$$

Assumption 3 : $\frac{\xi}{1+\xi}\left[1 + \frac{\lambda}{R(1-\tau_t)}\right] < 1/2$.

This assumption ensures that the agent choose a length of education for less than the half of his first part of life. It will also be useful later for the dynamic of the economy.

Firms

We assume that there is a small open economy. The capital is perfectly mobile so that the interest rate is determined at the international level. We also assume that this interest rate is constant over time at a level r. The production function has the following form[16] :

$$Y_t = AK_t^{\alpha}H_t^{1-\alpha} = AK_t^{\alpha}(N_t h_t(1-e_t) + z_{t-1}N_{t-1}h_{t-1})^{1-\alpha} \tag{6}$$

At the equilibrium of the firm we have :

$$F'_K(K,H) = \alpha A K_t^{\alpha-1}H_t^{1-\alpha} = r \tag{7}$$

Since r is a constant we can determine a relation between K and H. We have :

$$K_t = \left(\frac{\alpha A}{r}\right)^{\frac{1}{1-\alpha}} H_t \tag{8}$$

Wages per efficiency unit are :

$$w_t = (1-\alpha)AK_t^{\alpha}H_t^{-\alpha} = (1-\alpha)A\left(\frac{\alpha A}{r}\right)^{\frac{\alpha}{1-\alpha}} \equiv \nu \tag{9}$$

The production function can also be expressed as :

$$Y_t = A\left(\frac{\alpha A}{r}\right)^{\frac{\alpha}{1-\alpha}} H_t \tag{10}$$

Government

We assume that the government levy a tax. The level of this tax is assumed to be fixed exogenously at τ. This tax can be used to finance two kinds of public spending. The first is public education spending. The second is spending for the pension system. Government budget constraint has the following form :

$$G_t + \lambda(T - \underline{z})h_{t-1}w_t N_{t-1} = \tau w_t[N_t(1-e_t)h_t + \underline{z}h_{t-1}N_{t-1}] \tag{11}$$

As we saw in the previous section, λ is constant and fixed exogenously. Finally we obtain :

$$G_t = \nu[\tau N_t(1-e_t)h_t - \lambda T h_{t-1}N_{t-1}] \tag{12}$$

Giving the form of the public spending, expenditures for education are endogenously determined, that is to say that it is computed as a residue.

The dynamic of the model

[16]This form implies that the growth rate of the GDP per capita is : $\frac{\frac{Y_{t+1}}{N_{t+1}+TN_t}}{\frac{Y_t}{N_t+TN_{t-1}}} = \frac{Y_{t+1}}{Y_t}\frac{1}{1+n} = \frac{h_{t+1}}{h_t} = g$ on the growth path with $\underline{z} = 0$.

We now present the dynamic structure of the model and study its properties.

Definition 2 : The economy has a steady state growth path if h_t and $\tilde{y}_t$[17] grow at the same and constant growth rate g, and if e_t is constant.

As e_t only depends on T, so for a giving T, e_t will be constant. The growth rate of the economy is then given by the following expression (See appendix 1 for more details) :

$$g^{\frac{1+\xi}{\xi}} = \nu e(T)\left[g\frac{\tau(1-e(T))}{1+n} - \frac{\lambda T}{(1+n)^2}\right] \tag{13}$$

The left hand side is called $LHS(g)$, while the right hand side is called $RHS(g)$.

Proposition 4 : There exist a unique stable growth path $\forall T \in (0,1)$ if and only if we have $RHS(\hat{g}) > LHS(\hat{g})$, with $\hat{g} = \left(\frac{\xi}{1+\xi}\nu e(T)\frac{\tau(1-e(T))}{1+n}\right)^{\xi}$.

Proof : See appendix 1.

Lemma 2 : We also have $g > 1$ $\forall T \in (0,1)$ if and only if ν is sufficiently large.

Proof : See Appendix 1.

These conditions ensure that there exists an equilibrium in this economy and that it is characterized by a growth rate greater than one, and by a time dedicated to education that is bounded by zero and one. But what is interesting is to know how evolves theses economic variables when there is a demographic change, and notably an increase of life expectancy.

Proposition 5 : $\tilde{g}_{T=0} < \hat{g}_{T=0}$ and $LHS(\tilde{g}_{T=1}) > RHS(\tilde{g}_{T=1})$ are two sufficient conditions for the existence of a $\tilde{T}$ below which we have $\frac{\partial g(T)}{\partial T} > 0$ and above which $\frac{\partial g(T)}{\partial T} < 0$, with :

$$\tilde{g}_{T=1} = \frac{\frac{2\lambda}{R(1+n)(1-\tau)} + \frac{1}{1+n}}{\frac{\tau}{R(1-\tau)}\left[1 - 2\frac{\xi}{1+\xi}\left(1 + \frac{\lambda}{R(1-\tau)}\right)\right]} \tag{14}$$

and

$$\tilde{g}_{T=0} = \frac{R(1-\tau)(1+\xi)}{\tau(1+n)(1-\xi)} \tag{15}$$

[17] GDP per capita.

Proof : See Appendix 2.

In this model the existence of the pension system is essential. Indeed, giving our assumption 3, if $\lambda = 0$, we find that $\tilde{g} = 0$ for all T, that is to say that an increase of life expectancy always have a positive effect on the growth rate. Indeed, the fact that $e(1-e)$ is an increasing function of T means that the direct effect of e on the human capital accumulation is greater than the financing effect $(1-e)$ which is such that the increasing time dedicated to education decreases the working time and so on has a negative impact on the total amount of taxes.
When we introduce the PAYG pension system, there is too an impact on the amount that is used to finance such a system since the increasing life expectancy raises the time during which a pension is received. This has a negative effect on education spending, and so on the growth rate. We show that while life expectancy is not too high the pension system has not a significant impact on the growth rate. But, once life expectancy becomes larger, the pension system effect becomes the more important, and the growth rate is a decreasing function of life expectancy.

The other relevant demographic variable is the growth rate of the population (n). It reflects the evolution of the fertility rate.

Proposition 6 : An increase of the fertility rate has first a negative effect on the growth rate as long as $T \leq \hat{T}$, but it has positive effect on it for $T \geq \hat{T}$ if $LHS(\tilde{g}_2) > RHS(\tilde{g}_2)$ for $T = 1$.

Proof : See appendix 3.

An increase of the fertility rate has two opposite effects. It first decrease education spending per child that has a negative impact on the growth rate. But, it permits too to decrease spending per capita for the pension system. This increase education spending and has a positive effect on the growth rate. Finally, when life expectancy is low, the first effect dominates. But when life expectancy becomes greater, this is the second effect that becomes the most important.

4 Conclusion

This paper exhibits new channels that can explain the inverted U-shaped relationship between life expectancy and the growth rate of GDP per capita. But the main explanation is the impact of the PAYG pension system. We showed in a first time, with a physical capital model, that this pension system can counterbalance the positive effect of saving once life expectancy becomes high because the tax rate necessary to finance it increases greatly. In a second time, with a human capital model, we showed that the increasing life expectancy can increase the growth rate since people spend more time for education. But once life expectancy becomes high, spending for the pension system reduce too greatly education spending that has a negative effect on the growth rate of the human capital per capita.

We mentioned in the third section the impact of an aging society on education spending for some countries like the United States or for Switzerland. It would be interesting for futur research to show if we find it for other European countries.

5 Appendix

Appendix 1 :

Using equation (2) we obtain :

$$g_t^{1/\xi} = \nu \left[\frac{\tau N_{t-1}(1-e_{t-1})h_{t-1} - \lambda T h_{t-2} N_{t-2}}{N_t h_{t-1}} \right] e_t \tag{16}$$

Along the equilibrium growth path we have :

$$g^{\frac{1+\xi}{\xi}} = \nu e(T) \left[g \frac{\tau(1-e(T))}{1+n} - \frac{\lambda T}{(1+n)^2} \right] \tag{17}$$

The LHS is a convex function in g, while the RHS is a linear equation in g. Furthermore RHS cut the Y-axis at a negative value. Consequently if equilibrium exist there are two. But thanks to equation (16), we show easily that the higher steady state growth rate is the only stable, that is why we only concentrate on this value.

A necessary and sufficient condition for the existence of steady state growth rate of the economy is that at the point where the derivatives of LHS and RHS are equal we have RHS>LHS. This condition is expressed in the proposition 4.

We now have to prove that $g > 1, \forall T \in (0,1)$. It is sufficient to prove that RHS(g=1)>LHS(g=1). We obtain the following condition :

$$LHS_2(T) \equiv \nu e(T)\left[\frac{\tau(1-e(T))}{1+n}\right] > 1 + \frac{\nu\lambda T e(T)}{(1+n)^2} \equiv RHS_2(T) \tag{18}$$

We check easily that $LHS_2(T)$ is a concave function of T, while the $RHS_2(T)$ is a convex function of T. Therefore we only have to prove that $LHS_2(0) > RHS_2(0)$ and that $LHS_2(1) > RHS_2(1)$. For the first case we obtain that :

$$\nu > \tfrac{(1+\xi)^2(1+n)}{\tau\xi}$$

And for the second :

$$\nu\tfrac{\xi}{1+\xi}\left(1+\tfrac{\lambda}{R(1-\tau)}\right)\left(\tfrac{\tau}{1+n}\left(1-\tfrac{\xi}{1+\xi}\left(1+\tfrac{\lambda}{R(1-\tau)}\right)\right)-\tfrac{\lambda}{(1+n)^2}\right) > 1$$

A necessary condition for this equation is true is that :

$$1-\tfrac{\xi}{1+\xi}\left(1+\tfrac{\lambda}{R(1-\tau)}\right) > \tfrac{\lambda}{(1+n)\tau}$$

This only can be true if τ is sufficiently greater than λ.

Appendix 2 :

As the slope of the RHS of (17) increases with T and as the intersection of the RHS with the Y-axis decreases with T, we have to find the intersection of the old straight line with the new, that is to say after the increase of life expectancy. If at this intersection defined at a point $\tilde{g}$ we have that the LHS is greater than the RHS then the increase of life expectancy decreases the growth rate. Let us now determine $\tilde{g}$ for a giving T. Assume initial life expectancy be T, and that this life expectancy becomes T' with $T' = T + dt$ ($dt \to 0$). Let us also call part of equation (17) by : $RHS(g,T) = A(T)g - B(T)$, with $A(T) = \nu e(T)\frac{\tau(1-e(T))}{1+n}$, $B(T) = \nu e(T)\frac{\lambda T}{(1+n)^2}$, $A'(T) > 0$ and $B'(T) > 0$. $\tilde{g}$ is such that :

$$RHS(\tilde{g},T) = RHS(\tilde{g},T')$$

or,

$$\tilde{g}(A(T') - A(T)) = B(T') - B(T)$$
$$\Leftrightarrow \tilde{g} = \tfrac{(B(T')-B(T))/dt}{(A(T')-A(T))/dt}$$

We recognize derivatives of $B(T)$ and of $A(T)$that are respectively :

$$\frac{\partial B(T)}{\partial T} = \frac{\nu}{(1+n)^2}\lambda\frac{\xi}{1+\xi}\left(1+2\lambda\frac{T}{R(1-\tau)}\right)$$
$$\frac{\partial A(T)}{\partial T} = \nu\frac{\tau}{1+n}\frac{\xi}{1+\xi}\frac{\lambda}{R(1-\tau)}\left[\frac{1-\xi}{1+\xi}-2\frac{\xi}{1+\xi}\frac{\lambda}{R(1-\tau)}T\right]$$

Finally we have :

$$\tilde{g}_T = \frac{\frac{1}{1+n}\left(1+2\lambda\frac{T}{R(1-\tau)}\right)}{\frac{\tau}{R(1-\tau)}\left[\frac{1-\xi}{1+\xi}-2\frac{\xi}{1+\xi}\frac{\lambda}{R(1-\tau)}T\right]} \tag{19}$$

We obtain :

$$\tilde{g}_{T=0} = \frac{R(1-\tau)(1+\xi)}{\tau(1-\xi)(1+n)}$$
$$\tilde{g}_{T=1} = \frac{\frac{1}{1+n}\left(1+2\lambda\frac{1}{R(1-\tau)}\right)}{\frac{\tau}{R(1-\tau)}\left[\frac{1-\xi}{1+\xi}-2\frac{\xi}{1+\xi}\frac{\lambda}{R(1-\tau)}\right]}$$

As $\tilde{g}_T$ is an increasing function of T, for g is first an increasing function of T and then a decreasing function of it, we only have to prove that straight line after the increasing life expectancy cut the old while T is sufficiently small at a value $\tilde{g}$ such that $\tilde{g}_{T=0} < \hat{g}_{T=0}$. And for T sufficiently large we have to observe $LHS(\tilde{g}) > RHS(\tilde{g})$.

Appendix 3 :

The principle is the same as this of the appendix 2 as an increase of n reduces the slope of the RHS of (17) but increases the intersection of the straight line with the Y-axis. Therefore we have to find the intersection of the new straight line with the old and to know if at the intersection point, the RHS is under or over the LHS. In the first case ($RHS < LHS$), we know that the increase of the fertility rate has a positive effect on the growth rate. In the second case, we know that the equilibrium growth rate is lower.

Assume that the growth rate that was n becomes n' with $n' = n + dn$ and $dn \to 0$. We search the intersection point of the two RHS, that is to say for a given T :

$$\tilde{g}_2\frac{\tau(1-e(T))}{1+n} - \frac{\lambda T}{(1+n)^2} = \tilde{g}_2\frac{\tau(1-e(T))}{1+n'} - \frac{\lambda T}{(1+n')^2}$$

Using the same methodology as before we obtain :

$$\tilde{g}_2 = \frac{\lambda T}{(1-e(T))\tau}\frac{2}{(1+n)} \tag{20}$$

We see immediately that $\tilde{g}_2$ is an increasing function of T. For low values of T, for example for $T \to 0$, we always have that an increase of the fertility rate has a negative effect on growth since only the dilution of education spending prevails. For n has a positive effect on the growth rate we have to check that for high values of T ($T = 1$ for example) the financing effect of the pension system dominates. So we have to check that for $\tilde{g}_2^{T=1}$ we have : $LHS(\tilde{g}_2^{T=1}) > RHS(\tilde{g}_2^{T=1})$.

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