# The spenders-hoarders theory of capital accumulation, wealth distribution and fiscal policy

Pierre Pestieau[§] and Emmanuel Thibault[¶]

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**Abstract:** This paper proposes a simple and realistic OLG model which is easier to reconcile with the essential facts about consumer behavior, capital accumulation and wealth distribution, and to yield some new and surprising conclusions about fiscal policy. By considering a society in which individuals are distinguished according to two characteristics, altruism and wealth preference, we show that those who hold in the long run the bulk of private capital are not so much motivated by dynastic altruism as by a preference for wealth. Two types of social segmentation can result with different influences on the way fiscal tools influence wealth distribution across individuals. To a large extent our results seem to fit reality better than those obtained with standard optimal growth models for which dynastic altruism (or rate of patience) is the single source of heterogeneity.

**Keywords:** Altruism, Preference for wealth, Capital accumulation, Wealth distribution, Ricardian equivalence.

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[§]University of Liège, CORE, PSE and CEPR.

[¶]GREMAQ - Toulouse School of Economics and University of Perpignan.

"The love of wealth is therefore to be traced, as either a principal or accessory motive, at the bottom of all that the Americans do."

*Alexis de Tocqueville 1841.*

# 1 Introduction

The model of optimal capital accumulation with infinitely lived agents developed by Ramsey (1928) is one of the most popular models in macroeconomics. As infinitely lived agents are now often reinterpreted as dynasties of altruists (see Barro, 1974), it forms the core of many models of economic growth and it is extensively used for analyzing the effects of government policy.

In Barro-Ramsey framework, it is well known that all on the capital would end up in the hands of the most patient (or alternatively, the most altruist) households in a competitive equilibrium.[1] Before Ramsey (1928), as noted by Boyd (2000), the work of Rae (1834) and Fisher (1930) also suggests that the most patient households will accumulate all of the capital.

Importantly, this result seems to be robust since it is valid as soon as there exists some agents with infinite horizon (infinitely lived agents or unconstrained altruistic agents). Indeed, in societies with at least an agent *à la* Barro-Ramsey, the steady state is always[2] Pareto optimal and the long-run capital stock, driven by the degree of patience (or altruism) of the most patients (or altruists), is only held by the most patient (or altruist) agents.

The main goal of this paper is to upset these cornerstones results in optimal growth theory by constructing a simple and realistic overlapping generations model which is easier to reconcile with the essential facts about consumer behavior, capital accumulation and wealth distribution, and to yield some new and surprising conclusions about fiscal policy. Then, our paper can be viewed as an alternative modeling of dynamic wealth distribution that is generally dealt with using calibrated versions of stochastic growth models[3] or using theoretical models with imperfect credit market[4].

[1]This property has already been conjectured by Ramsey (1928) and it has been formally proved by Rader (1971), by Becker (1980) and Becker and Foias (1987) for the case in which households face borrowing constraints, and by Bewley (1982) for the case without borrowing constraints.

[2]See Muller and Woodford (1988), Nourry and Venditti (2001) or Thibault (2005).

[3]An elegant way to generate non-degenerate wealth distribution consists of introducing some idiosyncratic uninsurable risks in a growth model and calibrate it to fit data. Calibrated economies with Barro-Ramsey households have been for example studied by Aiyagari (1994), Castaneda, Diaz-Giménez and Rios-Rull (1998, 2003) and Quadrini (2000) when agents have identical preferences or by Krusell and Smith (1998) when agents differ regarding time discount rate.

[4]These standard theoretical models on wealth accumulation and on wealth distribution (for example, Banerjee and Newman, 1991, Galor and Zeira, 1993, Aghion and Bolton, 1997, Piketty, 1997, or Matsuyama, 2000, 2006) generally have three main ingredients: imperfect capital market, exogenous prices and warm-glow altruism. This form of altruism (joy of giving) is the most tractable, but it implies, contrary to the data, that the wealth held by an individual always has an inherited component.

*1.1. Why we need a new model ?*

Recently, Mankiw (2000) exhibit three pieces of evidence which suggest that we need a new macroeconomic model of fiscal policy. Indeed, the two canonical macrodynamic models – namely the Barro-Ramsey model with infinite horizon and the standard OLG model (due to Samuelson, 1958, and Diamond, 1965) with finite horizon – are inconsistent with the empirical finding that consumption tracks current income and with the numerous households with near zero wealth. In addition, the Diamond-Samuelson model is inconsistent with great importance of bequests in aggregate wealth accumulation. Then according to Mankiw (2000, p. 121): "A new model of fiscal policy needs a particular sort of heterogeneity. It should include both low-wealth households who fail to smooth consumption over time and high-wealth households who smooth consumption not only from year to year but also from generation to generation. That is, we need a model in which some consumers plan ahead for themselves and their descendants, while others live paycheck to paycheck."

From these observations, macroeconomists have focused on a new distinction to segment society in a dual way, that between spenders and savers, which echoes that introduced some time ago by Ramsey (1928) between people with high and low impatience and more recently that between altruists and non altruists.[5] The gist of these later distinctions is that savers, patient agents or altruistic households, end up accumulating wealth for the sake of transmission to their children whereas spenders, impatient agents or non altruistic households, don't save at all and if they do so, they do it for their own future consumption.

The question one can raise at this point is whether such a simplistic representation of society bears any resemblance to reality. In others words, le degré d'altruisme est il le paramètre clef de l'accumulation de richesse ? C'est sans nul doute un paramètre important puisque sans motif de transmission une richesse dynastique ne peut se transmettre et perdurer. Quelle que soit la richesse gigantesque de Warren Buffett ou de Bill Gates leurs choix respectifs de ne laisser rien ou peu à leurs enfants ne favorise pas l'accumulation de richesse de leurs lignées. Force est de constater that it is highly rare for a family fortune to last more than three generations, so much so that there's a well known adage: "Shirtsleeves to shirtsleeves in three generations". That is been to different cultures worldwide, but all versions convey the same message: the first generation makes the money, the second coasts along on it and the third erodes it completely.

[5]Among les modèles de type "savers-spenders", popularisés par Mankiw (2000), nous pouvons citer les travaux de Michel and Pestieau (1998, 1999, 2005) qui analysent les effets de politiques telles que la mise en place d'un payg system ou de l'estate taxation aussi bien dans des cadres où l'hétérogénéité porte seulement sur le degré d'altruisme (1998 pour le cas d'une offre de travail exogène, 1999 pour le cas d'une offre de travail endogène) ou porte à la fois sur le degré d'altruisme et l'abilité des agents (2005). Dans des cadres de type savers-spenders, Smetters (1999) analyse la robustesse de l'équivalence ricardienne, Nourry and Venditti (2001) étudient la stabilité et la détermination de l'équilibre de long-terme, Laitner (2001) try to explain secular changes in wealth inequality and inheritance in the US and UK data and Thibault (2005) try to explain the emergence of rentiers (i.e., of capitalists).

In the real life societies, individuals differ in many respects including altruism, but at the same time we observe that those who control capital accumulation are not particularly altruistic. For instance, Arrondel and Laferrère (1998) who distinguish very wealthy and just wealthy in France, show that for the former altruism plays a much minor role than for the latter. More recently De Nardi (2004) and Reiter (2004) developed general equilibrium models calibrated on US and/or Sweden data. They show that altruism cannot explain the top tail of the wealth distribution. To do it, they resort to what Carroll (2000) calls "a capitalist spirit motive". Based on the fact that households with higher levels of lifetime income have higher lifetime saving rates, Carroll (2000)'s paper argues that "the saving behavior of the (American) richest households cannot be explained by models in which the only purpose of wealth accumulation is to finance future consumption, either their own or that of heirs." Caroll concludes that to explain such a behavior one has to assume that some consumers regard the accumulation as a end in itself or as channel leading to power which is equivalent to assume that wealth is intrinsically desirable, what we call here "preference for wealth".

In a nutshell it seems that those who hold the bulk of private wealth are not so much motivated by dynastic altruism as by a preference for wealth. In other words, the key source of heterogeneity would not be only impatience or altruism but preference for holding wealth. In this paper, we look at this issue by considering a society in which individuals or rather dynasties are distinguished according to these two characteristics, altruism and wealth preference.

*1.2. An overview of our main contributions.*

Nous anticipons maintenant la suite du papier en décrivant et en soulignant la pertinence des principaux résultats que nous obtenons by considering an OLG economy with individuals differing in altruism and in preference for wealth. All individuals are altruistic: those with preference for wealth are labeled "hoarders-altruists"; those without such preference are either weak or strong altruists (see Table 1). Depending on these parameters, two kinds of equilibrium and segmentation can emerge. In the first type, denoted Equilibrium I, only the hoarders-altruists leave a positive bequest. In the second type of equilibrium, denoted Equilibrium II, only the weak altruists don't bequeath.

| | **Individual preferences** | | **Wealth transmission** | |
|---|---|---|---|---|
| **Types of individuals** | *Degree of Altruism* $\gamma$ | *Wealth Preference* $\delta$ | *Equilibrium I (two-class)* | *Equilibrium II (three-class)* |
| *Weak altruists* | weak | nil | nil (spenders) | nil (spenders) |
| *Strong altruists* | strong | nil | nil (spenders) | positive (savers) |
| *Hoarders-altruists* | positive | positive | positive (hoarders) | positive (hoarders) |

Table 1: Social segmentation

Concernant l'accumulation du capital, nos résultats diffèrent selon les deux types d'équilibres. Lorsque le degré d'altruisme des strong altruists est trop faible, le stock de capital peut alors être en sur-accumulation par rapport au stock de capital de la Golden Rule (hereafter G.R.) c'est à dire que l'equilibrium I peut être non Pareto-optimal; et ceux même si des agents mus par un altruisme dynastique (les hoarders-altruists) laissent un héritage positif. Ainsi le résultat des modèles de type "savers-spenders" selon lequel il suffit qu'un agent mu par de l'altruisme dynastique laisse un héritage pour avoir une économie dynamiquement efficace ne résiste pas à l'introduction de hoarders-altruists. Ceci peut expliquer pourquoi, bien que ces membres soient considérés comme altruistes, certaines sociétés peuvent être en sur-accumulation de capital. Lorsque l'altruisme des strong altruists est suffisamment élevé, le stock de capital correspond à celui de Modified Golden Rule (hereafter M.G.R.) par ce degré d'altruisme (Equilibrium II). Un tel résultat souligne la robustesse d'un des résultats obtenus par les modèles de type "savers-spenders": le stock de capital à long terme reste déterminé par les altruistes les plus patients.

C'est concernant la distribution de richesse que nos résultats sont les plus intéressants. The dynasties that bequeath are in a strong position in the way capital is shared. In the two-class equilibrium, the hoarders impose their view and in the three-class equilibrium, both hoarders and savers impose theirs. Ainsi, bien qu'à la M.G.R. ceux sont les plus patients qui déterminent le stock de capital de l'économie, ceux ne sont pas forcément eux, surprisingly enough, qui détiennent ce capital. En effet, il n'y a plus (comme dans les économies de type "savers-spenders") une seule mais plusieurs dynasties qui détiennent le capital: la dynastie de strong altruists et toutes les dynasties de hoarders.[6] Interestingly, non seulement la distribution de richesse n'est plus dégénérée (un seul point) mais elle est capable de rendre compte de ce que l'on observe dans la réalité puisque chaque famille ayant à la fois une préférence pour la richesse et un désir dynastique détient une part de la richesse accumulée. Importantly, we show that it possible that few hoarders with low degree of altruism detain more capital than numerous savers with high degree of altruism.

As the relation between wealth holding and altruism is not as simple as thought previously, we obtain new and surprising conclusions about fiscal policy. Commençons par résumer les principaux effects of a pay-as-you-go (hereafter PAYG) pension system or a public debt. At Equilibrium I, theses policies reduce the accumulation of capital and can improve or worsen both the welfare of the spenders and the one of the hoarders.

[6]We are not the first to exhibit theoretical variants of the Barro-Ramsey model in which the long-run distribution of wealth can be non-degenerate. For example, Epstein and Hynes (1983) or Lucas and Stokey (1984) show that there may exist stationary equilibria in which all households own positive amounts of capital when preferences are described by recursive utility. Sarte (1997) establishes that progressive taxation as another reason for the existence of stationary equilibria with a non-degenerate wealth distribution. A non-degenerate wealth distribution is also obtained by Dutta and Michel (1998) in a setting with imperfect altruism and linear price, by Boyd (2000) in an endogenous growth framework with learning-by-doing or by Falk and Stark (2001) with other form of altruism. Recently, Sorger (2002) shows, in the case where a government levies a progressive income tax, there exist infinitely many stationary equilibria in which all households own positive capital stocks. However, contrary to these papers, we are the first to obtain a distribution of wealth non-degenerate in a simple framework with logarithmic (and not recursive) utility function and without taxation and in which the standard M.G.R. capital stock is definable and holds.

At the M.G.R. (Equilibrium II), these policies have no effect on the stationary capital stock. However, as note Table 2, they increase the share of wealth held by the savers but decrease the one held by the spenders and the hoarders.

| | **Spenders** | **Savers** | **Hoarders** |
|---|---|---|---|
| **Share of capital held** | *Decreased* | *Increased* | *Decreased* |
| **Welfare** | *Decreased* | *Increased* | *Decreased* |

*Table 2: Effects of a PAYG pension system or a public debt at the MGR equilibrium.*

Ainsi, dans les modèles de type "savers-spenders" l'intervention de l'Etat a l'air injuste puisqu'elle transférent des ressources des spenders vers les savers. Introducing hoarders nous permet de montrer que l'intervention de l'Etat est loin d'être immoral puisqu'elle permet de transférer des ressources de toutes les dynasties avec de la préférence pour la richesse (et donc "des riches" et "des très riches") vers les autres. Note also that a PAYG pension system or a public debt have an opposite effect on unconstrained altruists whether there exists a preference for wealth (even very weak) or not. These policies improve the welfare of the savers but worsen the one of the spenders and the hoarders.

Les résultats concernant l'introduction d'une estate taxation sont eux aussi nouveaux et riches d'enseignements. At the M.G.R. equilibrium (see Table 3), an estate taxation reduces the accumulation of capital. Importantly, it increases the share of wealth held by spenders and hoarders at the expense of the share held by savers. Consequently, if the government wants too hurt the savers, it can introduces estate taxation; if it wants to favor them, it can use a PAYG pension or a public debt.

| | **Spenders** | **Savers** | **Hoarders** |
|---|---|---|---|
| **Share of capital held** | *Increased* | *Decreased* | *Increased* |
| **Welfare** | *Decreased* | *Decreased* | *Increased if $\delta^{HO}$ sufficiently large*<br>*Decreased if $\delta^{HO}$ sufficiently low* |

*Table 3: Effects of the estate taxation at the MGR equilibrium.*

Estate taxation is clearly a questionable instrument of redistribution: it hurts the wealthy, but favors the top wealthy. Moreover, estate taxation worsens the welfare both of the spenders and the savers but increases (decreases) the one of the hoarders if the degree of altruism of the savers is sufficiently high (low). We found here a new reason to deal with estate taxation with caution. If the objective of such a tax is to fight top wealth holding, we show that society might be better off without it.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 is devoted to the long-run capital accumulation and the long run wealth distribution. Section 4 studies the incidence of public debt, social security and estate taxation. A final section concludes. Proofs are gathered in Appendix.

## 2 The model

Consider a perfectly competitive economy which extends over infinite discrete time periods. The economy consists of $N \geq 1$ dynasties denoted by $h \in \{1, ..., N\}$. In each period $t$, the size of each dynasty $h$ is denoted by $N_t^h$ and grows at rate $n$. Total population size is $N_t$. We denote by $p_t^h$ the positive relative size of each dynasty $h$. It is time invariant. Hence, we have $N_t^h/N_t = p_t^h = p^h$, $\sum_{h=1}^{h=N} p^h = 1$ and $N_{t+1}/N_t = N_{t+1}^h/N_t^h = 1 + n$.

Individuals of a dynasty $h$ are identical within as well as across generations and live for two periods. All dynasties are made of altruistic individuals. We adopt Barro (1974)'s definition of altruism: parents care about their children welfare by including their children's utility in their own utility function and possibly leaving them a bequest. When young, altruists of dynasty $h$, born at time $t$, receive a bequest $x_t^h$, work during their first period (inelastic labor supply), receive the market wage $w_t$, consume $c_t^h$ and save $s_t^h$. When old, they consume $d_{t+1}^h$ a part of the proceeds of their savings and bequeath the remainder $(1+n)x_{t+1}^h$ to their $(1+n)$ children. Agents perfectly foresee the interest factor $R_{t+1}$. Bequest is restricted to be non-negative, which is an important assumption. We denote by $V_t^h$ the utility of an altruist of dynasty $h$:

$$
\begin{aligned}
V_t^h(x_t^h) = \max_{c_t^h, s_t^h, d_{t+1}^h, x_{t+1}^h} \quad & \ln c_t^h + \beta \ln d_{t+1}^h + \delta^h \ln x_{t+1}^h + \gamma^h \; V_{t+1}^h(x_{t+1}^h) \\
s.t \quad & w_t + x_t^h = c_t^h + s_t^h && (1) \\
& R_{t+1} s_t^h = d_{t+1}^h + (1+n) x_{t+1}^h && (2) \\
& x_{t+1}^h \geq 0
\end{aligned}
$$

where $V_{t+1}^h(x_{t+1}^h)$ denotes the utility of a representative child who inherits $x_{t+1}^h$. Parameter $\delta^h \geq 0$ measures the preference for wealth, $\gamma^h \in [0, 1)$ is the intergenerational degree of altruism of the dynasty $h$ and $\beta \in (0, 1]$ is the factor of time preference.

Note that, contrary to Barro (1974), our log-linear life-cycle utility is not restricted to depend only on life-cycle consumption. Indeed, the agent enjoys accumulating wealth for itself when $\delta^h > 0$. In this economy, the heterogeneity comes from the two parameters triggering saving (besides old age consumption): the preference for wealth $\delta^h$ and the degree of altruism $\gamma^h$. Then, each dynasty $h$ can be characterized by a pair $(\delta^h, \gamma^h) \in \mathbb{R}_+ \times [0, 1)$. From each pair $(\delta^h, \gamma^h)$ we can define the key parameter $\bar{\gamma}^h$ as follows:

$$\bar{\gamma}^h = \frac{\gamma^h(1+\beta) + \delta^h}{1 + \beta + \delta^h} \geqslant \gamma^h.$$

This parameter represents a "modified degree of altruism" which is larger or equal to $\gamma^h$. Indeed, when $\delta^h = 0$ we have $\bar{\gamma}^h = \gamma^h$ whereas $\bar{\gamma}^h > \gamma^h$ as soon as $\delta^h > 0$. This modified degree of altruism modified by the wealth preference allows us to segment, without loss of generality, the $N > 0$ dynasties as follows:

We first have $M$ dynasties ($M > 0$) which have no preference for wealth (i.e., $\delta^h = 0$) and are labeled from $h = 1$ to $M$. We assume that $\gamma^M \in (0, 1)$ and (if $M > 1$)

$\gamma^h \in [0, \gamma^M)$ for $h \in \{0, ..., M-1\}$. Therefore, $M$ is the most altruistic dynasty among the dynasties which don't have any preference for accumulating wealth. By convention, dynasties 1 to $M-1$ are considered as dynasties of weak altruists (W.A.) whereas the dynasty $M$ is a dynasty of strong altruists (S.A.).

We then have $N-M$ dynasties ($N > M$) which have some preference for wealth (i.e., $\delta^h > 0$) and are indexed $h = M+1, \ldots, N$. We assume that $\gamma^h \in (0,1)$ and (if $N > M+1$) $\bar{\gamma}^h \in [0, \bar{\gamma}^N)$ for $h \in \{M, ..., N-1\}$. Therefore, $N$ is the dynasty with the higher modified degree of altruism among the dynasties with preference for wealth. In our terminology these $N-M$ dynasties are dynasties of hoarders-altruists (H.A.).

Maximizing $V_t^h(x_t^h)$ subject to (1) and (2) gives the following first order conditions:

$$\forall h \in \{0, N\} \qquad d_{t+1}^h = \beta R_{t+1} c_t^h \tag{3}$$

$$\forall h \leq M \qquad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\gamma^h}{c_{t+1}^h} \leq 0 \quad (= \text{if } x_{t+1}^h > 0) \tag{4}$$

$$\forall h > M \qquad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\delta^h}{x_{t+1}^h} + \frac{\gamma^h}{c_{t+1}^h} = 0 \tag{5}$$

Unlike the $M$ dynasties without wealth preference, the optimal bequests of the $N-M$ dynasties with wealth preference are necessarily positive at all date, and hence, in the long run.

We can also write the saving of a given dynasty $h$. Indeed, we have:

$$s_t^h = \frac{1}{1+\beta}\Big[\beta(w_t + x_t^h) + \phi(R_{t+1})x_{t+1}^h\Big] \tag{6}$$

where $\phi(R) = (1+n)/R$ can be interpreted as a dynastic discount factor. Thus the higher the inheritance $x_t^h$ and earning $w_t$, the higher is saving. Saving increases also with intended bequest $x_{t+1}^h$.

Let us now turn to the production side. Production technology is represented by a Cobb-Douglas function with two inputs, capital $K_t$ and labor $L_t$ i.e., $Y_t = F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}$ with $\alpha \in (0,1)$ and $A > 0$. Homogeneity of degree one allows us to write output per worker as a function of the capital/labor ratio per worker, $Y_t/L_t = F(k_t, 1) = f(k_t) = Ak_t^\alpha$ with $k_t = K_t/L_t$, the capital/labor ratio.

Markets are perfectly competitive. Assuming, without loss of generality, that capital fully depreciates after one period, each factor is paid its marginal product:

$$w_t = f(k_t) - k_t f'(k_t) = A(1-\alpha)k_t^\alpha \quad \text{and} \quad R_t = f'(k_t) = A\alpha k_t^{\alpha-1} \tag{7}$$

In each period, the labor market clears, i.e., $L_t = N_t$ and the capital stock at time $t+1$ is financed by the savings of the young generation born in $t$. Hence we have

$K_{t+1} = N_t s_t$ with $s_t = \sum_{h=1}^{h=N} p^h s_t^h$. Therefore, in intensive form:

$$(1+n)k_{t+1} = \sum_{h=1}^{h=N} p^h s_t^h \tag{8}$$

We finish this section by three remarks. First, our approach comprises a wide class of OLG models.[7] Second, we do not make any assumption on the sign of $\bar{\gamma}^N - \gamma^M$. Finally, note that, according to Weil (1987) or Thibault (2000), the Barro's (1974) model with our Cobb-Douglas specification exhibits positive bequest if and only if $\gamma^M > \varepsilon \equiv \beta(1-\alpha)/[\alpha(1+\beta)]$. Even if, for the sake of generality, we allow for $\varepsilon > 1$, we have to keep in mind that $\varepsilon > 1$ corresponds to the case where the Barro's (1974) model has constrained altruists.

# 3 Capital accumulation and wealth distribution

In this section, we restrict our analysis to the steady states. We first study the long run behavior of hoarders-altruists. We have seen that their bequests $x_t^h$ are positive at each date $t$. Then, according to equations (3) and (5) we obtain in the steady-state denoted by subscript $\star$:

$$\forall h > M \qquad c_\star^h = \frac{\phi(R_\star) - \gamma^h}{\delta^h} x_\star^h \quad \text{and} \quad d_\star^h = \beta R_\star c_\star^h \tag{9}$$

Merging (1), (2) and (9) we obtain the (positive) level of stationary bequest of a dynasty $h$ of hoarders-altruists:

$$\forall h > M \qquad x_\star^h = \frac{\bar{\delta}^h w_\star}{\phi(R_\star) - \bar{\gamma}^h} \tag{10}$$

where $\bar{\delta}^h \equiv \delta^h/(1+\beta+\delta^h)$. This term is the relative weight of wealth in the life-cycle utility $\ln c_t^h + \beta \ln d_{t+1}^h + \delta^h \ln x_{t+1}^h$.

Since, the bequest of hoarders-altruists is positive, the steady-state $\phi(R_\star)$ necessarily satisfies $\bar{\gamma}^N < \phi(R_\star)$ and, according to (6) and (10), stationary saving $s_\star^h = x_\star^h + w_\star - c_\star^h$ of a dynasty $h$ of hoarders-altruists is given by:

$$\forall h > M \qquad s_\star^h = \frac{1}{1+\beta}\left(\beta + \frac{\bar{\delta}^h\left[\beta + \phi(R_\star)\right]}{\phi(R_\star) - \bar{\gamma}^h}\right) w_\star \tag{11}$$

We now turn to the dynasties with no wealth preference. According to equations (3) and (4), the long-term behavior of each of them must satisfy:

$$\forall h \leq M \qquad \gamma^h \leq \phi(R_\star) \quad (= \text{if } x_\star^h > 0) \tag{12}$$

[7] When $N = M = 1$ and $\gamma^1 \leq \varepsilon$ we obtain the Diamond's (1965) model (or equivalently the Barro's (1974) model with constrained altruists). When $N = M = 1$ and $\gamma^1 > \varepsilon$ we have Barro's (1974) model with positive bequests. When $N = M > 1$ and $\gamma^M > \varepsilon$, we obtain the Barro's (1974) model with heterogenous dynasties studied by Vidal (1996). When $N = M = 2$, $\gamma^1 = 0$ and $\gamma^2 > \varepsilon$ our economy is similar to that studied by Michel and Pestieau (1998), Mankiw (2000) or Nourry and Venditti (2001).

Hence, among these $M$ dynasties, only the strong altruists, i.e., the dynasty $M$ with the highest degree of altruism, has the possibility to leave a bequest. Indeed, if there exists a dynasty $m \in \{1, ..., M-1\}$ such that $x_\star^m > 0$ then equation (12) is not satisfied for dynasties $h$ where $h \in \{m+1, ..., M\}$. Then, the weak altruists are constrained altruists in the long run and their saving $s_\star^h$ is such that:

$$\forall h \leq M-1 \qquad s_\star^h = \frac{\beta}{1+\beta} w_\star \tag{13}$$

Things are more complicated for the behavior of strong altruists. However, according to (12), if $x_\star^M$ is positive then the steady state capital stock $k_\star$ is equal to that of the M.G.R. capital stock (i.e., $k_\star = f'^{-1}[(1+n)/\gamma^M]$) and we have:

$$k = \left[\frac{\alpha A \gamma^M}{1+n}\right]^{\frac{1}{1-\alpha}} \equiv k_\star^M, \;\; R = \frac{1+n}{\gamma^M} \equiv R_\star^M, \;\; w = A(1-\alpha)\left[\frac{\alpha A \gamma^M}{1+n}\right]^{\frac{\alpha}{1-\alpha}} \equiv w_\star^M$$

For the altruists without wealth preference we follow Mankiw (2000) by labeling as spenders the dynasties of constrained altruists and as savers the dynasties of unconstrained altruists. Since the hoarders-altruists wish to accumulate capital for its own sake we label them as hoarders. Then, to sum up our economy (see Table 1), we have shown that the weak altruists are always spenders and the hoarders-altruists are always hoarders. The case of the strong altruists is ambiguous: they are spenders when $x_\star^M = 0$ and savers when $x_\star^M$ is positive. As a consequence, the bequest motive of the strong altruists determines the long run wealth distribution of the society.

Knowing the saving function of each dynasty allows us to characterize the long run capital accumulation in our economy. Indeed, using (6), (11) and (13) we can rewrite the equation of capital accumulation (8) according to whether or not the strong altruists leave a bequest.

When $x_\star^M = 0$, according to (11), (13), and using the fact that $(1+n)k/w = \alpha\phi(R)/(1-\alpha)$, the equilibrium condition (8) is equivalent to:

$$\digamma(\phi(R_\star)) \equiv \frac{\beta(\phi(R_\star) - \varepsilon)}{\varepsilon\left(\phi(R_\star) + \beta\right)} - \sum_{h=M+1}^{N} \frac{p^h \bar{\delta}^h}{\phi(R_\star) - \bar{\gamma}^h} = 0 \tag{14}$$

This equation determines the capital stock of the economy when the strong altruists are spenders.

When $x_\star^M$ is positive, we have $\phi(R^M) = \gamma^M$ and according to (6), (11) and (13), the equilibrium condition (8) gives:

$$x_\star^M = \left[\frac{\beta(\gamma^M - \varepsilon)}{\varepsilon(\gamma^M + \beta)} - \sum_{h=M+1}^{N} \frac{p^h \bar{\delta}^h}{\gamma^M - \bar{\gamma}^h}\right] \frac{w_\star^M}{p^M} = \digamma(\gamma^M) \times \frac{w_\star^M}{p^M} \tag{15}$$

This equation determines the bequest left by the strong altruists in the steady-state. Accordingly, the strong altruists are savers or spenders according to whether or not $\digamma(\gamma^M)$ is positive.

From equations (14) and (15) we can now study both the existence of the steady state and the long run capital accumulation of our economy.

**Proposition 1** - THE LONG RUN CAPITAL ACCUMULATION.

*a – When strong altruists are insufficiently altruist, the steady state is a spenders-hoarders equilibrium where the stationary capital stock $k_\star$ is the solution of* (14).

*This capital stock increases with the proportion, the degree of altruism and the wealth preference of the hoarders but is independent of the degree of altruism of the spenders.*

*The stationary capital stock $k_\star$ is in under-accumulation, at the G.R. or in over-accumulation of capital if respectively $\sum_{h=M+1}^{N} p^h \delta^h/(1-\gamma^h)$ is smaller than, equal to or larger than $\beta\,(1-\varepsilon)\,/\varepsilon$.*

*b – When strong altruists are sufficiently altruist, the steady state is a savers-spenders-hoarders equilibrium where the stationary capital stock is equal to that of the M.G.R. capital stock $k_\star^M$.*

*This capital stock increases with the degree of altruism of the savers but is independent of the proportion of each dynasty, and of the degree of altruism of the spenders and the hoarders.*

*The stationary capital stock $k_\star^M$ is below the G.R. level.*

PROOF – See Appendix A. □

In Appendix A, we exhibit a threshold value $\tilde{\gamma}$ (satisfying $\tilde{\gamma} \geq \bar{\gamma}^N$) of the degree of altruism of strong altruism below which the strong altruists are insufficiently altruist and, consequently, spenders and above which they are sufficiently altruist and, consequently, savers.

The uniqueness of the stationary equilibrium is standard given the Cobb-Douglas specification. It is however interesting to observe that even in this very simple setting we end up with an endogenously segmented society.

The present paper generalizes the results of the "savers-spenders" models in several respects. It presents a more realistic setting. Also, the introduction of agents with two different characteristics (altruism and preference for wealth) allows for testing the robustness of those models. Indeed, the mere introduction of at least one dynasty of hoarders sizeably affects the equilibrium and the resulting stratification.

Two patterns are now possible in the long term. In the first, we have a two-class society with spenders and hoarders that has never been studied in the literature. The spenders belong both to the weak and strong altruists whereas the hoarders belong to hoarders-altruists. The second pattern generalizes the "savers-spenders" models by introducing a third class: the hoarders. We now have a society of savers, spenders and hoarders since in this pattern the strong altruists are savers.

We observe that the three-class result does not hold anymore when the hoarders have a modified degree of altruism, $\bar{\gamma}^h$, that is high enough relatively to the others. Indeed, it suffices that $\bar{\gamma}^N > \gamma^M$ for not having savers, in other words, for having the strong altruists bequeathing nothing. This does not mean that $\gamma^N > \gamma^M$. A low factor of altruism $\gamma^N$ is compatible with a high $\bar{\gamma}^N$. The degree of altruism $\gamma^M$ of strong altruists is nevertheless fundamental to determine which kind of equilibrium we end up with.

If it is to bequeath, the strong altruists needs to have a degree of altruism sufficiently high, i.e., such that $\gamma^M > \bar{\gamma}^N$. Note that the value of $\tilde{\gamma}$ is independent of the degree of altruism and of the proportion of the spenders. On the contrary this threshold value $\tilde{\gamma}$ depends on the proportion and the degree of altruism of the $N - M$ dynasties of the hoarders. All things being equal the higher the proportion, the degree of altruism and the wealth preference of the hoarders, the higher is the threshold $\tilde{\gamma}$ and the lower is the likelihood to have a three-class equilibrium that vindicates the results of Michel and Pestieau (1998) and Mankiw (2000).

Importantly, the macrodynamic properties of the equilibrium spenders-hoarders are very different from those of the equilibrium savers-spenders-hoarders. When strong altruists are savers, the economy is at the M.G.R. steady state which depends on the degree of altruism $\gamma^M$ of savers, but not on their proportion. Note that this result is similar to those of Kaldorian models (see, for instance, Kaldor, 1956, Pasinetti, 1962, or Britto, 1972) but it is obtained in an endogenous way. As well-known since Ramsey (1928) and Becker (1980), the most patients (or altruists) impose their view on the long-run capital accumulation, whatever their size. Then, our savers-spenders-hoarders equilibrium seems to be equivalent to the equilibrium obtained in the "savers-spenders" models with heterogenous agents with no wealth preference. Il est cependant important de remarquer que contrairement aux modèles de type "savers-spenders" et à l'intuition[8] de Ramsey (1928), le M.G.R. equilibrium n'a plus à long terme deux catégories d'individus mais trois: savers, spenders and hoarders. L'équilibre ayant deux catégories d'agents, les spenders et les hoarders, converge lui vers un stock de capital $k_\star$ supérieur à celui $k_\star^M$ de la M.G.R.

At the microeconomics level the introduction of altruists-hoarders has an important implication. Even though the dynasty of strong altruists imposes its view on capital accumulation, it is not the only one to bequeath and hold wealth. The hoarders are also bequeathing even though they can have a negligible degree of altruism. When instead we have a spenders-hoarders equilibrium, the stock of capital is not any more determined by a single dynasty. Now the stock of capital depends on both the degree of altruism and on the preference for wealth of the $N - M$ dynasties of hoarders and on the proportion of each of them.

The fact that the logic of capital accumulation is totally different in the two equi-

[8]Even though optimal growth theorists and Ramsey in particular are not concerned by intragenerational issue, he notes in his 1928's seminal paper that if agents differ in their time preference society will be split in two classes: "the thrifty enjoying bliss and the improvident at the subsistence level".

libriums is reinforced by our results on the dynamic efficiency of the two steady-states. These results also emphasize the influence of wealth preference. Even though all agents have some altruism for their children, the competitive equilibrium can be dynamically inefficient (and thus non Pareto optimal) when some agents have some preference for wealth per se. Indeed at the spenders-hoarders equilibrium, the higher the proportion and the wealth preference of a dynasty of hoarders, the higher is the possibility of over-accumulation. Conversely, the higher the degree of altruism of a dynasty of hoarders, the higher is the possibility of under-accumulation. Thus the main result of Barro (1974) that dynastic altruism implies dynamic efficiency does not resist to the introduction of agents having some preference for wealth not just as a means of saving for retirement or for bequests but just for itself. Intuitively it is clear that with such a taste for wealth the stock of capital is likely to be higher than when there is no such a taste.

Things are different in the three-class equilibrium. The stock of capital being consistent with the M.G.R., the economy is dynamically efficient irrespectively of the presence of hoarders. To sum up, depending on the fundamentals, the introduction of hoarders may have an impact or not on the accumulation of capital. Note that the introduction of hoarders is not necessarily negative. Even when it increases the capital stock, it can be efficient as long as the G.R. level is not overtaken.

To illustrate numerically the possibility of a dynamically inefficient equilibrium even if all agents are altruistic, we now consider an economy in which the population consists of a proportion $p_1$ of weak altruists with degree of altruism $\gamma_1 < \varepsilon$, a proportion $p_2$ of strong altruists with degree of altruism $\gamma_2 > \varepsilon$ and a proportion $p_3$ of hoarders with degree of altruism $\gamma_3$. We adopt the following scenario: $\gamma_2 = 0.9$, $\gamma_3 = 0.89$, $\beta = 0.5$, $\alpha = 1/3$, $A = 1$ and $n = 0$. Clearly hoarders is very altruistic. On Figure 1, we see how the steady-state stock of capital changes with $\delta$ and increasing values of $p_3$. The G.R. level of capital is given by the horizontal dotted line. The M.G.R. is given by the horizontal segment of the continuous line.

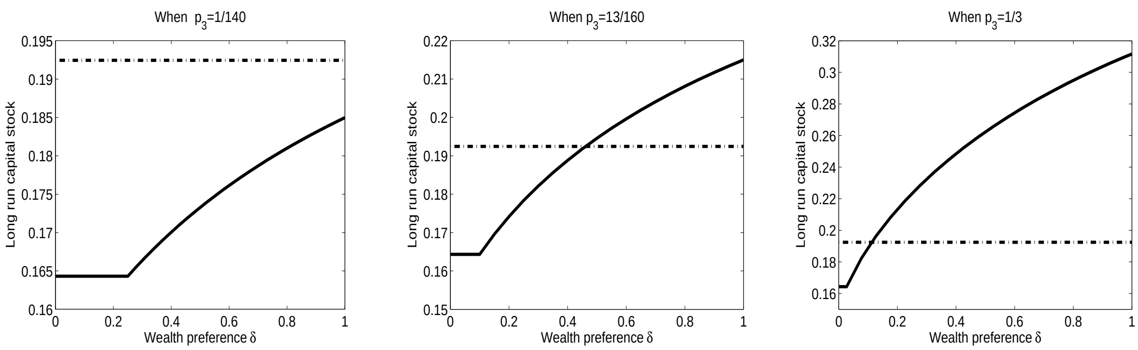


Figure 1: Capital accumulation and dynamic efficiency

When the proportion of hoarders is very low (1/140) the economy follows the M.G.R. as long as $\delta \leqslant 0.25$. When $\delta > 0.25$ we have a two-class equilibrium but the level of capital stock stays dynamically efficient. In this particular case, increasing wealth preference can be viewed as socially desirable. When the proportion of hoarders is a bit higher, we have the three-class equilibrium for low values of

$\delta$ ($\delta = 0.1$ for $p_3 = 13/160$). When $\delta$ increases, we move to the two-class equilibrium but also to dynamically inefficient capital stocks. For $p_3 = 1/3$ the economy turns dynamically inefficient for $\delta = 0.1$. This shows how Barro (1974)'s intuition that dynastic altruism leads to dynamic efficiency is not robust to a slight introduction of preference for wealth.

We now turn to the way capital is held by the different agents. So doing we obtain a better grasp of the long run distribution of wealth in the two alternative equilibria. One of the motivations of this paper was to show that top wealth is not necessarily held by the strong altruists, which was the conclusion as soon as there exists some savers (infinitely lived agents or unconstrained altruistic agents).

Comparing to the savers-spenders literature, our model seems to be most relevant to study the long term distribution of wealth. In the earlier models relying on the single characteristic of either patience or altruism, the equilibrium wealth distribution was reduced to two points: positive wealth for the most patient or altruist, and zero wealth for the others. By introducing some preference for wealth and thus the category of hoarders we now have a more complex and realistic distribution of wealth. We now have $N - M + 1$ or $N - M$ types of wealth-holders. Indeed, it is straightforward that the dynasties which held long run wealth are the dynasties of bequeathers. We could get the share of wealth held by each of the $N$ dynasties. However to keep the analysis simple, we now focus on the share of each of the three types of dynasties: savers, spenders, hoarders. With this simple presentation, we now study the comparative statics of this wealth-holding.

**Proposition 2** - THE LONG RUN WEALTH DISTRIBUTION.

*a – At the spenders-hoarders equilibrium, the higher the proportion, the degree of altruism and the preference for wealth of hoarders, the higher is the fraction* $\mu^{HO}$ *of the stationary capital* $k_\star$ *held by the hoarders and the lower is the fraction* $\mu^{SP}$ *held by the spenders.*

*b – At the savers-spenders-hoarders equilibrium, the higher the degree of altruism of the savers, the higher is the fraction* $\nu^{SA}$ *of the stationary capital* $k_\star^M$ *held by the savers and the lower are* $\nu^{HO}$ *and* $\nu^{SP}$ *the fractions respectively held by the hoarders and the spenders. The higher is the proportion, the degree of altruism and the preference for wealth of a dynasty of hoarders, the higher is* $\nu^{HO}$.

PROOF – See Appendix B. □

Whatever the equilibrium regime, the spenders hold some capital that is related to saving for retirement. The distribution of wealth in the equilibrium with spenders and hoarders depends on the stock of capital, which is not the case of the three-class equilibrium. The larger the capital stock, the higher is the share held by the hoarders. Wealth sharing in the two-class equilibrium is more or less imposed by the behavior of hoarders. Both the overall stock of capital and the share they hold increase with their degree of altruism and their taste for wealth. Conversely, the share of wealth held by the spenders is independent of their own characteristics. Intuitively, one could say that the hoarders get the additional capital accumulation they generate $\left(k_\star - k_\star^M\right)$.

In the three-class equilibrium, things are clearly different. Sharing now depends on the degree of altruism of both savers and hoarders. The more altruistic the savers, the higher is the capital stock $k_\star^M$, and the higher is their share. When the degree of altruism of the savers increases, $\nu^{HO}/\nu^{SP}$ decreases and, thus, even if the shares of both savers and hoarders diminish, the hoarders loose more that the spenders. When the factor of altruism of a dynasty of hoarders increases, then the share and the amount of capital held by the hoarders increase at the only expenses of savers. In other words, variation is the degree of altruism of hoarders have not impact on the share of capital held by the spenders.

Our findings are summarized in the following table.

| | **Variations of capital sharing** | | | | |
|---|---|---|---|---|---|
| | EQUILIBRIUM I *(two-class)* | | EQUILIBRIUM II *(three-class)* | | |
| | **Spenders** $\mu^{SP}$ | **Hoarders** $\mu^{HO}$ | **Spenders** $\nu^{SP}$ | **Savers** $\nu^{SA}$ | **Hoarders** $\nu^{HO}$ |
| **Increase of Parameter** | | | | | |
| Altruism Degree $\gamma$ of W.A. | *Unchanged* | *Unchanged* | *Unchanged* | *Unchanged* | *Unchanged* |
| Altruism Degree $\gamma^M$ of S.A. | *Unchanged* | *Unchanged* | *Decreased* | *Increased* | *Decreased* |
| Altruism Degree $\gamma$ of H.A. | *Decreased* | *Increased* | *Unchanged* | *Decreased* | *Increased* |
| Wealth Preference $\delta$ of H.A. | *Decreased* | *Increased* | *Unchanged* | *Decreased* | *Increased* |

*Table 4: Changes in Wealth Distribution according to individual parameters.*

To sum up, the dynasties that bequeath are in a strong position in the way capital is shared. In the two-class equilibrium, the hoarders impose their view and in the three-class equilibrium, both hoarders and savers impose theirs. It is important to see the key role plaid by the preference for wealth. Whatever the final equilibrium the share of capital held by the hoarders increases with their preference for wealth.

To illustrate numerically the relevancy of our model we consider the three-type setting used previously for illustrate the possibility of dynamic inefficiency. We now introduce a scenario to illustrate the fact that the relation between wealth holding and altruism is not as simple as thought previously. In this scenario we choose: $\gamma_2 = 0.8$, $\gamma_3 = 0.5$, $\delta = 0.5$, $\beta = 0.5$, $\alpha = 1/3$, $A = 1$ and $n = 0$.

As soon as $p_3 > 0.06$, we have a two-class equilibrium. We see that as $p_3$ increases beyond this threshold value the share of capital held by the hoarders increase from a share 0.225 ($p_3 = 0.06$) to 1 ($p_3 = 1$). Consequently, even though hoarders are less altruistic than the strong altruists, they end up holding relatively more wealth. In this

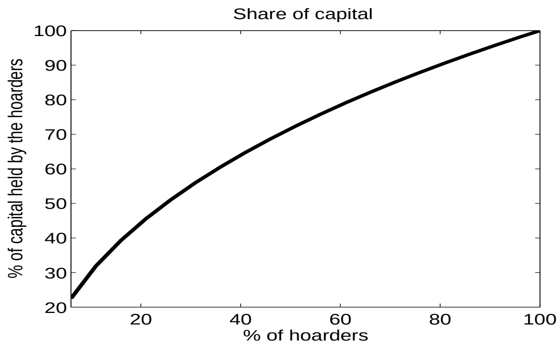


Figure 2: Share of capital held by the hoarders (Scenario 2)

example the hoarders hold almost 25% of total wealth even though they are poorly altruistic and only represent 5% of total population.

# 4 Alternative fiscal policies

We now turn to alternative fiscal policies such as PAYG pension system, public debt and estate taxation. We want to see how the results obtained in optimal growth models with spenders and/or savers change or can be extended with the introduction of the hoarders.

## 4.1 Pay-as-you-go pensions

Our PAYG pension system consists of a payroll levy, $\tau_t$, paid in period $t$ by workers and a pension benefit, $\theta_t$, paid to the retirees of generation $t-1$ so that $\theta_t = (1+n)\,\tau_t$. Then, the budget constraints (1) and (2) become:

$$w_t + x_t^h - \tau_t = c_t^h + s_t^h \quad \text{and} \quad R_{t+1} s_t^h + \theta_{t+1} = d_{t+1}^h + (1+n)\, x_{t+1}^h \tag{16}$$

We begin our study of the incidence of PAYG pension system in our economy with a fraction of hoarders by focusing both on the savings and the bequests behavior of each types of individuals.

Obviously, the spenders do not leave bequest. Then, after calculus, their savings and their bequests satisfy:

$$\forall h \leq M-1 \quad s_\star^h(\tau) = \frac{\beta w_\star(\tau) - [\beta + \phi_\star(\tau)]\tau}{1+\beta} \qquad \text{and} \qquad x_\star^h(\tau) = 0 \tag{17}$$

Concerning the hoarders, they always bequeath and the equilibrium conditions (9) always holds. Using $\tilde{\omega}_\star(\tau) = w_\star(\tau) - [1-\phi_\star(\tau)]\tau$ the part of the life-cycle income $\Omega_\star^h(\tau)$ independent of $h$, their savings and their bequests is such that:

$$\forall h > M \quad s_\star^h(\tau) = \frac{\zeta_\star^h(\tau) w_\star(\tau)}{1+\beta} - \frac{\tau[\beta + \phi_\star(\tau)]\chi_\star^h(\tau)}{1+\beta} \quad \text{and} \quad x_\star^h(\tau) = \frac{\bar{\delta}^h \tilde{\omega}_\star(\tau)}{\phi_\star(\tau) - \bar{\gamma}^h} \tag{18}$$

where:
$$\zeta_\star^h(.) = \beta + \frac{\bar{\delta}^h[\beta + \phi_\star(.)]}{\phi_\star(.) - \bar{\gamma}^h} \qquad \text{and} \qquad \chi_\star^h(.) = 1 + \frac{\bar{\delta}^h[1 - \phi_\star(.)]}{\phi_\star(.) - \bar{\gamma}^h}$$

Finally, the strong altruists are savers if $x_\star^M(\tau)$ is positive and are spenders if $x_\star^M(\tau)$ is nil. Then, we now distinguish these two cases to determine how the PAYG pension system affect the capital accumulation.

When strong altruists are spenders, using both (17) and (18), we can rewrite (8)to obtain at the equilibrium:

$$\digamma\left[\phi_\star(\tau)\right] = -\frac{\tau}{w_\star(\tau)}\vartheta[\phi_\star(\tau)] \quad \text{with} \quad \vartheta[\phi_\star(.)] = 1 + \sum_{h=M+1}^{N} \frac{p^h\bar{\delta}^h[1 - \phi_\star(.)]}{\phi_\star(.) - \bar{\gamma}^h} \tag{19}$$

Importantly, even if $\phi_\star(.)$ can be greater than one, the function $\vartheta[\phi_\star(.)]$ is always positive. Indeed, $\vartheta[\phi_\star(.)]$ is a decreasing function of $\phi_\star(.)$ such that $\lim_{\phi_\star(.)\to+\infty}\vartheta[\phi_\star(.)]=$ $1 - \sum_{h=1}^{+\infty} p^h\bar{\delta}^h > 0$. Note that, when $\tau = 0$, we recoup (14).

When strong altruists are savers, the economy is at the M.G.R. Indeed, (12) holds becauses the first order conditions (3) and (4) are not modified by the PAYG pension system. Then, the equality (8) allows us to obtain, after some tedious computations, the bequests of savers:

$$x_\star^M(\tau) = \digamma(\gamma^M) \times \frac{w^M}{p^M} + \frac{\tau}{p^M} \times \vartheta(\gamma^M) \tag{20}$$

Note that, when $\tau = 0$, we recoup (15).

From equations (19) and (20) we can now study the impact of a PAYG pension system both on the long run capital accumulation of the economy, the redistribution across the dynasties and the welfare of each dynasties.

**Proposition 3** THE EFFECTS OF A PAYG PENSION SYSTEM.

*a – At the spenders-hoarders equilibrium, the PAYG pension system reduces the accumulation of capital. The decrease in savings of the spenders is not compensated by the hoarders. Indeed, concerning both the bequest and the savings of the hoarders, the impact of the PAYG pension system is ambiguous.*

*The introduction of the PAYG pension system can improve or worsen both the welfare of the spenders and the one of the hoarders.*

*b – At the savers-spenders-hoarders equilibrium, the PAYG pension system has no effects on the accumulation of capital. The decrease in the saving both of the spenders and the hoarders is compensated by an increase of the savings of the savers. If the bequest of the spenders remains nil, the introduction of the PAYG pension system reduces the one of the hoarders but increases the one of the savers. This introduction* *increases the share of capital held by the savers but decreases the shares of capital held respectively by the spenders and the hoarders.*

*The introduction of the PAYG pension system improves the welfare of the savers but it worsens the welfare both of the spenders and the hoarders.*

Proof – See Appendix C. □

According to (20), the equilibrium is a three-class one if and only if $\tau > \tau^M \equiv -w^M F(\gamma^M)/\vartheta(\gamma^M)$. If without PAYG pension we have a three-class equilibrium (case where $F(\gamma^M) > 0$), this type of equilibrium remains so with PAYG regardless of the size of $\tau$. On the contrary, if the economy is in a two-class equilibrium without PAYG pensions (case where $F(\gamma^M) < 0$) there exists a payroll tax rate $\tau^M$ above which introducing a PAYG pension scheme will imply a three-class equilibrium. Then, the introduction of a PAYG pension system can imply a transition from a spenders-hoarders equilibrium to a spenders-savers-hoarders equilibrium. The opposite shift is not possible. The intuition underlying this possible transition is that in the two-class equilibrium introducing a PAYG pension leads to capital decumulation in the long run. This decline can lead to the capital level consistent with the M.G.R., for which we get a three-class equilibrium.

In the two class equilibrium, the higher $\tau$, the less do spenders save. The first effect induces a drop in the capital stock. Furthermore this effect can be offset or reinforced depending on the influence of the PAYG pension system on the level of bequests and saving by the hoarders. Concerning the bequests of the hoarders, we have (see Appendix C):

$$\forall h > M \qquad \frac{\partial x_\star^h(k_\star(\tau),\tau)}{\partial \tau} = \frac{\partial x_\star^h(k_\star(\tau),\tau)}{\partial k_\star(\tau)}\frac{\partial k_\star(\tau)}{\partial \tau} - \frac{\bar{\delta}^h[1-\phi_\star(\tau)]}{\phi_\star(\tau)-\bar{\gamma}^h}$$

When $\alpha$ and $\tau$ are sufficiently low we show that the first term of RHS is positive. Then, if the equilibrium value $k_\star(\tau)$ is low enough, hoarder's bequests increase with $\tau$. Intuitively, the introduction of the PAYG pension system leads to an increase in the second period income of the hoarders. This wealth gain in the second period induces the hoarders to increase their bequests. Note however that when the equilibrium is dynamically inefficient (i.e., $\phi[k_\star(\tau)] < 1$), the second term of the RHS is negative and the effect of the PAYG pension system on the hoarder's bequests is ambiguous. This effect can even be positive as the first term of the RHS can also be negative when $\tau$ and $\alpha$ are sufficiently high. Intuitively bequests by the hoarders can decrease as the PAYG pension system has a depressive effect on first-period income. Thus there exists a wealth loss in the first period that depresses bequests by the hoarders.

Concerning the saving of the hoarders we establish that it is reduced by the PAYG pension system when their bequests are themselves reduced. Conversely, note that the fact that the PAYG pension system increases the bequest of the hoarders is not sufficient to imply an increase in the saving of the hoarders. Intuitively, it exists two antagonist effects when $\tau$ increases. First, according to (6) an increase of the bequest

$x^h_\star = x^h_t = x^h_{t+1}$ imply an increase of the saving of the hoarders when the prices $w$ and $R$ are given. However, it also exists when $\tau$ increases a "general equilibrium effect" which leads to reduce the capital stock and, consequently, $w_\star$ and $\phi(R_\star)$. According to (6), when $x^h_\star = x^h_t = x^h_{t+1}$ is given this effect implies a decrease of the saving of the hoarders.

To sum up PAYG pension systems always generate a drop in saving by spenders. This decline can be accompanied by a drop in the saving of the hoarders thus reinforcing the decrease in the stationary capital stock. On the contrary, a PAYG scheme can also lead to an increase in the saving of hoarders. Yet this possible increase is too weak to compensate for the decline in the saving of spenders. In other words, PAYG pension systems are not macroeconomically neutral when the economy is in a two-class equilibrium at the outset.

Things are different when society is segmented in three classes at the start. In that case, the PAYG pension system is neutral given that the stationary stock of capital is consistent with the M.G.R. capital stock. This neutrality result generalizes those obtained by Barro (1974), Michel and Pestieau (1998) and Mankiw (2000). In that case, neutrality property obtained in the "savers-spenders" literature resists to the introduction of hoarders-altruists.

Even if the PAYG pension system has no effect on the stationary stock of capital, it modifies the long run wealth distribution because saving by the three classes is affected by the fiscal policy. That of the spenders and the hoarders decreases and that of the savers increases. Saving by the hoarders and the spenders does not equally react to a change in the payroll tax because bequests by the savers and the hoarders don't move in the same way. PAYG pension systems increase bequests by the savers while reducing those by the hoarders. Intuitively, the hoarders have a bequest motive that is related to wealth accumulation. Their bequests are thus a constant proportion of income $w^M_\star - (1-\gamma^M)\tau$, which decreases with $\tau$. Things are different for the savers as their bequests increase with $\tau$, given that $x^M_\star(\tau) = x^M_\star(0) + \tau\vartheta(\gamma^M)/p^M$. Following equation (8), the sum of all the bequests (i.e., $\sum_{h=1}^N x^h_\star$) is constant in the savers-spenders-hoarders equilibrium. Thus as $\tau$ is raised, increased bequests by the savers fully compensate the drop in bequests by the hoarders.

The fact that, contrary to the capital stock, wealth distribution is modified is already a result obtained by Michel and Pestieau (1998) or Mankiw (2000): they show that a PAYG pension system implies wealth redistribution from the spenders to the savers. Introducing hoarders extends those results: the fraction of capital they hold $\nu^{HO}(\tau)$ decreases with $\tau$. There is thus some redistribution from hoarders to savers. Puisqu'elle pénalise les super riches l'intervention de l'Etat qui semblait inequitable dans le cas des modèles de type "savers-spenders" devient ici un élément en défaveur des riches. The direction of redistribution between spenders and hoarders is ambiguous; it depends on the proportion of hoarders; it has to be high enough so that $\nu^{HO}(\tau) - \nu^{SP}(\tau)$ increases with $\tau$.

The effect of a PAYG pension system on individual welfare[9] depends on the type of equilibrium we are concerned with. In the two-class case, both $k_\star$ and $R_\star$ vary with $\tau$. Thus, the welfare of the spenders can be written as:

$$V_\star^{SP}(\tau) = (1+\beta)\ln\tilde{\omega}_\star(\tau) + \beta\ln R_\star(\tau) + cst$$

As $R_\star(\tau)$ increases with $\tau$ but $w_\star(\tau)$ decreases, the variations of $V_\star^{SP}(\tau)$ can be ambiguous. For low values of $\tau$, $\tilde{\omega}_\star(\tau)$ and $R_\star(\tau)$ vary in opposite directions. As $w'_\star(\tau)$ and $\phi'_\star(\tau)$ are both negative, $\tilde{\omega}_\star(\tau)$ is a decreasing function if we are in underaccumulation. On the contrary, if we are in overaccumulation, $\tilde{\omega}_\star(\tau)$ increases with $\tau$ and the PAYG pension system may increase the welfare of the spenders. Moreover, the welfare of the hoarders is given by:

$$V_\star^{HO}(\tau) = V_\star^{SP}(\tau) + \delta^h\ln\tilde{\omega}_\star(\tau) + (1+\beta)\ln\{\phi_\star(\tau) - \bar{\gamma}^h + \bar{\delta}^h[1-\phi_\star(\tau)]\}$$
$$-(1+\beta+\delta^h)\ln(\phi_\star(\tau) - \bar{\gamma}^h) + cst$$

The relation between $V_\star^{HO}(\tau)$ and $\tau$ is also ambiguous. However, when the preference for wealth is weak enough and if there is a lot of overaccumulation, a PAYG pension system increases the welfare of the hoarders. Henceforth, in overaccumulation, a PAYG pension system may be Pareto improving. After all, this is not a surprising result.

In the case of a three-class equilibrium the welfare incidence is easier as the stock of capital $k_\star^M$, and thus $R_\star^M$, don't depend on $\tau$. Thus, both the welfare of the spenders and the hoarders can be rewritten as:

$$V_\star^{SP}(\tau) = (1+\beta)\ln[w_\star^M - (1-\gamma^M)\tau] + cst \quad \text{and} \quad V_\star^{HO}(\tau) = (1+\beta+\delta^h)\ln[w_\star^M - (1-\gamma^M)\tau] + cst$$

Hence, one clearly sees that $\tau$ has a depressive effect on the welfare of these two types of dynasties. Turning to the savers, we have:

$$V_\star^{SA}(\tau) = \ln[w_\star^M + (1-\gamma^M)(x_\star^M(\tau) - \tau)] + cst$$

Since $x_\star^M(\tau) - \tau - x_\star^M(0) = \tau[\vartheta(\gamma^M)/p^M - 1] > 0$, one clearly sees that $\tau$ has a positive effect on the welfare of the savers.

We find here an extension of the key result of Michel and Pestieau (1998) and Mankiw (2000) according to whom a PAYG scheme improves the welfare of the savers while decreasing the welfare of the spenders. Their result resists to the introduction of hoarders but their are gloss over by the fact introducing a PAYG pension system increases the welfare of the savers but decreases the one of the hoarders; even if the hoarders has an infinitesimal preference for wealth and consequently if the hoarders is quasi similar to the savers.

[9] Given that $d_\star^h = \beta R_\star c_\star^h$ and $\Omega_\star^h = c_\star^h + d_\star^h/R_\star$, the long run welfare of a dynasty $h$ can be rewritten as $V_\star^h = (1+\beta)\ln\Omega_\star^h + \beta\ln R_\star + \delta^h\ln x_\star^h + cst$ where $\Omega_\star^h(\tau) = \tilde{\omega}_\star(\tau) + [1-\phi_\star(\tau)]x_\star^h(\tau)$.

## 4.2 Public debt

We now turn to the standard question of whether or not debt policy can be steady state welfare improving. In each period, the government faces the budget constraint $B_t = (1 + r_t)B_{t-1} - L_t T_t$, where $B_t$ is the total level of debt in $t$ and $T_t$ is a lump-sum tax paid by the working generation. We assume that the debt was used at time 0 to the benefits of the retirees. There is no other government spending. We write $b_t = B_{t-1}/N_t$ and assume that $b_t = b$ is constant. This yields $T_t = (r_t - n)b$.

With this public debt scheme, only two equations are changed. The first period budget constraint (1) is now:

$$w_t + x_t^h - T_t = c_t^h + s_t^h \tag{21}$$

and the relation linking capital and savings (8) becomes:

$$(1 + n)(k_{t+1} + b) = \sum_{h=1}^{h=N} p^h s_t^h \tag{22}$$

We begin our study of the incidence of public debt in our economy with a fraction of hoarders bu focusing focus both on the savings and the bequests behavior of each types of individuals. Obviously, the spenders do not leave bequest. Then, after calculus and using the first period income net of tax $b$, $\tilde{\omega}_\star(b) = w_\star(b) - [R_\star(b) - (1 + n)]b$, their savings and their bequests satisfy:

$$\forall h < M \quad s_\star^h(b) = \frac{\beta}{1+\beta}\tilde{\omega}_\star(b) \quad \text{and} \quad x_\star^h(b) = 0 \tag{23}$$

Concerning the hoarders, they always bequeath and the equilibrium conditions (9) always holds. Thus, after some computations, their bequests and their bequest is such that:

$$\forall h > M \quad s_\star^h(b) = \frac{\zeta_\star^h(b)\tilde{\omega}_\star(b)}{1+\beta} \quad \text{and} \quad x_\star^h(b) = \frac{\bar{\delta}^h \tilde{\omega}_\star(b)}{\phi_\star(b) - \bar{\gamma}^h} \tag{24}$$

where $\zeta_\star^h(.)$ and $\chi_\star^h(.)$ are defined in equation (18).

Finally, the strong altruists are savers if $x_\star^M(b)$ is positive and are spenders if $x_\star^M(b)$ is nil. We now distinguish these two cases to see how the public debt affects the capital accumulation.

When strong altruists are spenders, using the equilibrium condition (22) we obtain after some calculus:

$$\digamma\,[\phi_\star(b)] = -\frac{(1+n)b}{\phi_\star(b)w_\star(b)}\vartheta[\phi_\star(b)] \tag{25}$$

where $\vartheta[\phi_\star(.)]$ is defined in equation (19). Note that, when $b = 0$, we recoup (14).

When strong altruists are savers, the economy is at the M.G.R capital stock $k_\star^M$. Indeed, (12) holds because the first order conditions (3) and (4) are not modified by the public debt scheme. Then, the equation of capital accumulation (22) allows us to obtain, after some tedious computations, the bequests of the savers:

$$x_\star^M(b) \quad = \quad \digamma(\gamma^M) \times \frac{w_\star^M}{p^M} + \frac{(1+n)b}{\gamma^M p^M} \times \vartheta(\gamma^M) \tag{26}$$

Note that, when $b = 0$, we recoup (15).

From equations (25) and (26) we can now study the impact of the public debt both on the long run capital accumulation of the economy, the redistribution across the dynasties and the welfare of each dynasties.

**Proposition 4** THE EFFECTS OF A PUBLIC DEBT.

*a – At the spenders-hoarders equilibrium, the public debt always reduces the accumulation of capital. When the stationary capital stock without debt is in under accumulation of capital, then both the savings of the spenders and the bequests of the hoarders are reduced by the public debt. However, it is possible that both the savings of the spenders and the bequests of the hoarders are augmented when the stationary capital stock without debt is in over accumulation of capital. Concerning the savings of the hoarders, the impact of the public debt is ambiguous. Importantly, if the economy without debt is dynamically inefficient, it exists a constant public debt policy* $b^G$ *which restore the dynamic efficiency by leading the economy at the G.R. equilibrium.*

*The introduction of the public debt can improve or worsen both the welfare of the spenders and the one of the hoarders.*

*b – At the savers-spenders-hoarders equilibrium, the public debt has no effects on the capital accumulation. The decrease in the saving both of the spenders and the hoarders is compensated by an increase of the savings of the savers. If the bequest of the spenders remains nil, the introduction of the public debt reduces the one of the hoarders but increases the one of the savers. This introduction increases the share of wealth (capital plus bonds) held by the savers but decreases the shares of wealth held respectively by the spenders and the hoarders.*

*The introduction of the public debt improves the welfare of the savers but it worsens the welfare both of the spenders and the hoarders.*

PROOF – See Appendix D. □

As in the case of a PAYG pension system, the public debt can make the economy shift from a two-class equilibrium to a three-class equilibrium, the reverse being impossible. The reason is simple: a three-class equilibrium occurs if and only if $b > b^M \equiv -\digamma(\gamma^M)w^M\gamma^M/[(1+n)\vartheta(\gamma^M)]$. Then, if without debt the equilibrium is one with three classes (case where $\digamma(\gamma^M) > 0$), it remains so with debt and whatever the level of the debt. On the contrary if, without debt, the equilibrium is one with two classes (case where $\digamma(\gamma^M) < 0$), then there exists a level of debt $b^M$ above which public borrowing leads to a three-class equilibrium. This change of regime is made possible because in a two-class equilibrium public borrowing reduces steady-state capital accumulation. This reduction can be such that the steady-state capital stock converges to the M.G.R., which implies a three-class equilibrium.

At the two-class equilibrium, contrary to the PAYG pension system case, the saving of the spenders is not always reduced by public borrowing because it increases with $\tilde{\omega}_\star(b)$. Then, as capital stock is reduced (i.e, $k_\star(0) > k_\star(b)$), saving by the spenders is always reduced by the public debt when $R_\star(b) > 1+n$, i.e. when $k_\star(b)$ is below the G.R. level of capital. However, when $k_\star(b)$ is above the G.R. level saving by the spenders is reduced (resp: increased) by public borrowing if $W_\star(b)$ is larger (resp: lower) than one where $W_\star(b) = w_\star(0)/\tilde{\omega}_\star(b)$. According to (24), the bequest of the hoarders is always reduced by the public debt when $k_\star(b)$ (and consequently $k_\star(0)$) is in under-accumulation of capital. However, when $W_\star(b) < 1$ (in this case, the capital stock $k_\star(0)$ is necessarily in over-accumulation) the bequest of the hoarders is reduced (resp: augmented) by the public debt if $W_\star(b)$ is larger (resp: lower) than $\varsigma_\star(b) = (\phi_\star(0) - \bar{\gamma}^h)/(\phi_\star(b) - \bar{\gamma}^h) > 1$. Since $\zeta_\star'(b) > 0$, we have $\zeta_\star(b) > \zeta_\star(0)$. Then, the saving of the hoarders is augmented by the public debt when $W_\star(b) < 1$. When $W_\star(b) > 1$ this saving is augmented (resp: reduced) according to $\zeta_\star(b)$ is larger (resp:) than $\zeta_\star(0)W_\star(b)$.

To sump up the saving behavior that follows public borrowing in the two-class equilibrium, we distinguish among three cases. When $W_\star(b) > \zeta_\star(b)/\zeta_\star(0) > 1$, public borrowing reduces saving by both the spenders and the hoarders. When $\zeta_\star(b)/\zeta_\star(0) > W_\star(b) > 1$, saving by the spenders decreases and that by the hoarders increases following the introduction of the public debt. When $W_\star(b) < 1$, both savings increase. Note that this latter case is only operative if the equilibrium without debt is below the G.R. This case is surprising and impossible with PAYG pension systems. The reason is that aggregate private saving is not, contrary to the PAYG pension system case, equal to just the stock of capital, but to the stock of capital plus bonds. Thus in case (*iii*), $k_\star(b) + b$ increases with respect to $b$ even if $k_\star(b)$ decreases.

From an initial situation of overaccumulation, we show that there exists a level of debt $b^G \in (0, b^M)$ that leads to the G.R. capital stock. We thus find for this heterogeneous society the result obtained by Diamond (1965) for a society consisting only of spenders: public debt can lead to a Pareto optimal growth path. In such a society consisting only of spenders, if at the outset the economy is in under accumulation, the public debt is welfare worsening in the steady-state. With heterogeneous agents, this negative effect is mitigated because when $b$ reaches $b^M$ the economy switches to a three-class equilibrium in which the stock of capital corresponds to the M.G.R. and is invariant to the public debt.

Public debt has no macroeconomic effect on the three-class equilibrium. This neutrality result at the aggregate level strengthens the intuition that just one saver is enough to obtain Ricardian equivalence. Michel and Pestieau (1998) and Mankiw (2000) show that this result keeps holding with the introduction of spenders. Here we show that it resists to the further introduction of hoarders.

As in the case of PAYG pension systems, if the public debt is neutral in aggregate terms, it modifies wealth distribution because the saving of our three classes of individuals change. Saving by the spenders and the hoarders is reduced and that by the savers is increased. Saving by the hoarders and the spenders does not react equally to public borrowing because of bequests. Public debt increases bequests of the savers while reducing bequests of the hoarders. Hoarders leave a bequest that is a proportion of the income $\tilde{\omega}_\star^M(b) = w_\star^M - (1+n)(1/\gamma^M - 1)b$ which declines as $b$ increases. Things are different for the savers; their bequests increase with $b$ as $x_\star^M(b) = x_\star^M(0) + (1+n)b\vartheta(\gamma^M)/(\gamma^M p^M)$. However, the sum of all the bequests is an increasing function of $b$ in the savers-spenders-hoarders equilibrium. Hence, in contrast with the PAYG pension system case, the savers necessarily increase their bequests by an amount higher than what is necessary to compensate the decrease of bequests by the hoarders when $b$ increases.

The fact that, contrary to the capital stock, wealth distribution is modified is already a result already obtained for PAYG pension systems. The share of capital held by the savers increases with $b$ whereas the share held both by the spenders and the hoarders decreases. The direction of redistribution between the hoarders and the savers is ambiguous; it depends on the proportion of hoarders in society. Consequently, if the government wants too hurt the hoarders (i.e., the top wealthy) and to favor the savers, it can use a PAYG pension or a public debt.

Concerning individual welfare[10], as we now show, the distinction between two types of equilibrium is going to provide for the effect of the public debt results quite similar to those obtained for the effect of a PAYG pension system. At the spenders-hoarders equilibrium, we have for the welfare of the spenders:

$$V_\star^{SP}(b) = (1+\beta)\ln\tilde{\omega}_\star(b) + \beta\ln R_\star(b) + cst$$

As $R_\star(b)$ increases and $w_\star(b)$ decreases with $b$, the variations of $V_\star^{SP}(b)$ can be ambiguous. For low variations of $b$, $\tilde{\omega}_\star(b)$ and $R_\star(b)$ vary in opposite directions. As $w'_\star(b) - R'_\star(b)$ is negative, remark that $\tilde{\omega}_\star(b)$ decreases with $b$ when $\phi_\star(b) < 1$. On the contrary, $\tilde{\omega}_\star(b)$ increases with $b$ in case of overaccumulation and thus the public debt improves the welfare of the spenders. The welfare of the hoarders is given by:

$$\begin{aligned} V_\star^{HO}(b) = V_\star^{SP}(b) + \delta^h\ln\tilde{\omega}_\star(b) + (1+\beta)\ln[\phi_\star(b) - \bar{\gamma}^h + \bar{\delta}^h(1-\phi_\star(b))] \\ -(1+\beta+\delta^h)\ln(\phi_\star(b) - \bar{\gamma}^h) + cst \end{aligned}$$

The relation between $V_\star^{HO}(b)$ and $b$ are also ambiguous. However with weak preference for wealth and important overaccumulation introducing public debt increases the welfare of the hoarders. In that case, public debt is Pareto improving.

Moving to the three-class equilibrium, one writes the welfare of spenders as:

$$V_\star^{SP}(b) = (1+\beta)\ln[w_\star^M - (1/\gamma^M - 1)(1+n)b] + cst$$

[10]Given that $d_\star^h = \beta R_\star c_\star^h$ and $\Omega_\star^h = c_\star^h + d_\star^h/R_\star$, the long run welfare of a dynasty $h$ can be rewritten as $V_\star^h = (1+\beta)\ln\Omega_\star^h + \beta\ln R_\star + \delta^h\ln x_\star^h + cst$ where $\Omega_\star^h(b) = \tilde{\omega}_\star(b) + [1-\phi_\star(b)]x_\star^h(b)$.

As to the hoarders, their welfare is:

$$V_\star^{HO}(b) = (1+\beta+\delta^h)\ln[w_\star^M - (1/\gamma^M - 1)(1+n)b] + cst$$

Thus, the higher public debt, the lower the welfare of both the spenders and the hoarders is. Turning to the savers, we have:

$$V_\star^{SA}(b) = \ln[w_\star^M + (1-\gamma^M)(x_\star^M(b) - \frac{b(1+n)}{\gamma^M})] + cst$$

Since $x_\star^M(b) - b(1+n)/\gamma^M - x_\star^M(0) = b(1+n)[\vartheta(\gamma^M)/p^M - 1]/\gamma^M > 0$ and $\vartheta(\gamma^M) > 1$, introducing public borrowing increases the welfare of the savers.

To sum up, we have the same type of results as for the PAYG pension. Unconstrained altruists benefit from national debt whereas both the hoarders and the spenders welfare decreases.

## 4.3 Estate taxation

We now turn to a third instrument that is natural in a setting where bequests play such an important role: inheritance or estate taxation. Again, we focus on the steady state solution. The tax scheme is simple: an estate tax of fixed rate $\kappa \in [0,1)$ that is redistributed in each period $t$ in a lump-sum way in an amount $\theta_t$, the same for all. Hence, the revenue constraint is simply equivalent to:

$$\theta_t = \kappa \sum_{h=1}^{N} p^h x_t^h$$

Then, the first budget constraint (1) of an agent of dynasty $h$ becomes:

$$w_t + (1-\kappa)x_t^h + \theta_t = c_t^h + s_t^h \tag{27}$$

Moreover, given $\theta_t$, the optimal condition for saving (3) is unchanged but that for bequests both of the savers and the hoarders (4) and (5) are now distorted:

$$\forall h \leq M \qquad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\gamma^h(1-\kappa)}{c_{t+1}^h} \leq 0 \quad (= \text{if } x_{t+1}^h > 0) \tag{28}$$

$$\forall h > M \qquad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\delta^h}{x_{t+1}^h} + \frac{\gamma^h(1-\kappa)}{c_{t+1}^h} = 0 \tag{29}$$

We now want to see what is the incidence of estate taxation in our economy with a fraction of hoarders. In particular, we focus both on the savings and the bequests behavior of each types of individuals. Obviously, the spenders do not leave bequest. Then, after calculus and using the notation $\tilde{\omega}_\star(\kappa) = w_\star(\kappa) + \theta_\star(\kappa)$, their savings and their bequests satisfy:

$$\forall h < M \quad s_\star^h(\kappa) = \frac{\beta}{1+\beta}\tilde{\omega}_\star(\kappa) \quad \text{and} \quad x_\star^h(\kappa) = 0 \tag{30}$$

Concerning the hoarders, they always bequeath and (9) always holds. Thus, after computations, their savings and their bequests are such that:

$$\forall h < M \quad s_\star^h(\kappa) = \frac{\hat{\zeta}_\star^h(\kappa)\tilde{\omega}_\star(\kappa)}{1+\beta} \quad \text{and} \quad x_\star^h(\kappa) = \frac{\bar{\delta}^h \tilde{\omega}_\star(\kappa)}{\phi_\star(\kappa) - \bar{\gamma}^h(1-\kappa)} \tag{31}$$

where $\hat{\zeta}_\star^h(\kappa) = \bar{\delta}^h[\beta(1-\kappa) + \phi_\star(\kappa)]/[\phi_\star(\kappa) - \bar{\gamma}^h(1-\kappa)] + \beta$. Note that, larger is the redistribution $\theta_\star$, larger is the bequest of the hoarders whereas larger is the tax rate $\kappa$, lower are the bequests of the hoarders.

Finally, the strong altruists are savers if $x_\star^M(\kappa)$ is positive and are spenders if $x_\star^M(\kappa)$ is nil. We now distinguish these two cases to see how the estate taxation affects the capital accumulation.

When the strong altruists are spenders, we can use the equilibrium condition $(1 + n)k_\star(\kappa) = \sum_{h=1}^{M} p^h s_\star^h(\kappa)$ to obtain after some calculus:

$$\Lambda(\phi_\star(\kappa), \kappa) \equiv \frac{\beta(\phi_\star(\kappa) - \varepsilon)}{\varepsilon(\phi_\star(\kappa) + \beta) + \kappa\beta(\phi_\star(\kappa) - \varepsilon)} - \sum_{h=M+1}^{N} \frac{p^h \bar{\delta}^h}{\phi_\star(\kappa) - \bar{\gamma}^h(1-\kappa)} = 0 \tag{32}$$

When $\kappa = 0$, we recoup (14) since $\Lambda(\phi_\star(0), 0) = F(\phi_\star(0))$.

When the strong altruists are savers, according to (28) we have $\phi_\star(\kappa) = \gamma^M(1-\kappa)$. Then, contrary to the public debt scheme or the PAYG pension system, an estate tax modifies the stock of capital of the savers-spenders-hoarders equilibrium. Indeed, the M.G.R. capital stock is affected by the estate taxation since we have in long run:

$$k_\star^M(\kappa) = \left[\frac{\alpha A \gamma^M(1-\kappa)}{1+n}\right]^{\frac{1}{1-\alpha}}$$

Then, larger is the tax rate $\kappa$, lower is the long run capital stock $k_\star^M(\kappa)$. We can remark that this capital stock does not depend on the proportion of the savers, the spenders or the hoarders. In the no tax case, this capital stock is obviously the one of the M.G.R. Using the fact that $\phi_\star(\kappa) = \gamma^M(1-\kappa)$, equation (8) allows us to obtain, after some tedious computations, the bequests of the savers:

$$x_\star^M(\kappa) = \left\{\frac{\beta}{\varepsilon}\left[\frac{\gamma^M(1-\kappa) - \varepsilon}{\gamma^M + \beta + \kappa\gamma^M}\right] - \left[\frac{\gamma^M + \beta + \kappa\beta\gamma^M/\varepsilon}{\gamma^M + \beta + \kappa\gamma^M}\right] \sum_{h=M+1}^{N} \frac{p^h \bar{\delta}^h}{\gamma^M - \bar{\gamma}^h}\right\} \frac{w_\star^M(\kappa)}{p^M} \tag{33}$$

where $w_\star^M(\kappa) = A(1-\alpha)k_\star^M(\kappa)^\alpha$.

From equations (32) and (33) we can now study the impact of an estate taxation both on the existence of the steady states, the long run capital accumulation and the redistribution across the dynasties. Given the complexity of the problem at hand, we make two simplifications. First we take a tax reform viewpoint by focusing on an infinitesimal change in the tax rate at a zero level. Second we assume that $N = M + 1$; in other words, there is only one dynasty of hoarders.

**Proposition 5** The effects of the estate taxation.

*(a) At the spenders-hoarders equilibrium, with only one dynasty of hoarders, the estate tax reduces the capital accumulation. It also depresses the bequest of the hoarders. Its impact on savings is ambiguous. However, when the introduction of estate taxation increases (decreases) the savings of the hoarders (spenders), it also increases (decreases) the savings of the spenders (hoarders).*

*The introduction of the estate taxation can improve or worsen the welfare of both the spenders and the hoarders. Note that estate taxation improves (worsens) the welfare of the hoarders (spenders), it improves (worsens) also that of the spenders (hoarders).*

*(b) At the savers-spenders-hoarders equilibrium, estate taxation reduces the capital accumulation. It reduces the savings of all the agents. It also depresses the bequest of the savers but increases (decreases) that of the hoarders if the degree of altruism of the savers is sufficiently high (low). Moreover, the introduction of estate taxation decreases the share of wealth held by the savers but increases that held by the spenders and the hoarders.*

*The introduction of estate taxation worsens the welfare both of the spenders and the savers but increases (decreases) that of the hoarders if the degree of altruism of the savers or their preference for wealth are sufficiently high (low).*

Proof – See Appendix E. □

According to (33) we have a three-class equilibrium if and only if $\kappa < \kappa^M \equiv \varepsilon(\gamma^M + \beta)\Lambda(\gamma^M, 0)/[1 + \sum_{h=M+1}^{N} p^h \bar{\delta}^h/(\gamma^M - \bar{\gamma}^h)]$. Thus if, in the absence of estate tax, we have a two-class equilibrium $\mathcal{F}(\gamma^M) = \Lambda(\gamma^M, 0) < 0$, we keep this type of equilibrium with estate taxation. On the contrary, if we have a savers-spenders-hoarders equilibrium without estate taxation $\mathcal{F}(\gamma^M) = \Lambda(\gamma^M, 0) > 0$, there is a positive level of taxation $\kappa^M$ above which the equilibrium becomes a spenders-hoarders equilibrium. Hence, introducing an estate tax can lead to go from a three-class equilibrium to a two-class one; the other way around is not possible. Estate taxation has thus the opposite effect relative to public debt and PAYG pension.

What is the intuition of such a switch of regime? Public debt or PAYG pension system induces the strong altruists to increase their bequests and hence reinforces the portion of savers and may lead to a switch of regime from two- to three-classes. Estate taxation discourages bequeathing by the strong altruists and may lead to the disappearance of savers. Consequently, the only possible switch is that from three- to two-classes.

We now analyze the behavior of different agents in the two types of equilibrium. In the spenders-hoarders equilibrium, according to (30), saving by the spenders depends on $\tilde{\omega}_\star(\kappa)$. Then, according to Appendix D, there exists two opposite effects. The first one follows from the decrease of capital accumulation triggered by estate taxation; the second one is redistributive and follows from the lump-sum transfer financed by estate taxation. Concerning saving by the hoarders, beyond the two effects just mentioned, there is a third one due to changes in $\hat{\zeta}_\star^N(\kappa)$. This third effect is negative. Hence if estate taxation increases saving by the hoarders, then it also increases saving by the spenders. Conversely, if estate taxation depresses saving by the spenders, it also depresses saving by the hoarders. Turning to the level of bequests by the hoarders (one single dynasty) it can be expressed as $x_\star^N(\kappa) = \bar{\delta}^N w_\star(\kappa)/\{\phi_\star(\kappa) - \bar{\gamma}^N + \kappa(\bar{\gamma}^N - p^N \bar{\delta}^N)\}$. As the numerator decreases and the denominator increases with $\kappa$, the estate tax has a depressive effect on the bequests of the hoarders.

In the three-class equilibrium, estate taxation has quite different effects. First of all, let us remember that it has a depressive incidence on capital accumulation. This is a result that is consistent with that obtained by Mankiw (2000) and Michel and Pestieau (1998). As in the two-class equilibrium, saving varies with disposable income $\tilde{\omega}_\star(\kappa)$. However, $\tilde{\omega}_\star(\kappa)$ decreases with $\kappa$ when the rate of estate taxation is low and $\hat{\zeta}_\star^h(\kappa) = \beta + \bar{\delta}^h(\beta + \gamma^M)/(\gamma^M - \bar{\gamma}^h)$ does not depend on $\kappa$. Thus, according to (30) and (31), savings of both spenders and hoarders decrease when $\kappa$ decreases, contrary to what is happening in the two-class equilibrium.

As to the influence of $\kappa$ on saving by the savers, consider first the effect of $\kappa$ on their bequests described by (33). The higher the estate tax rate, the lower the bequests by the savers are. Since $(1+\beta)s^M(\kappa) = \beta\tilde{\omega}_\star(\kappa) + (1-\kappa)(\gamma^M + \beta)x_\star^M(\kappa)$, this negative effect of taxation on bequests has an impact on saving. The first term of the RHS decreases with $\kappa$ (for low $\kappa$) and the second term always decreases with $\kappa$. Consequently, estate taxation depresses saving by the savers.

To sum up, for low tax rates, saving by the three types of agents decreases and capital accumulation goes down unambiguously. This is in contrast with the neutral effect of either public debt or unfunded pensions.

Turning to the bequests of the hoarders, according to (31), there exists two opposite effects when $\kappa$ increases and calculations of Appendix E lead to:

$$\forall h > M \qquad \frac{\partial x_\star^h(\kappa)}{\partial \kappa}\bigg|_{\kappa=0} \gtrless 0 \quad \text{if and only if} \quad \gamma^M \gtrless \frac{\alpha\beta}{\alpha\beta + 1 - \alpha}.$$

Thus contrary to the bequests of the savers, the bequests of the hoarders don't necessarily decrease as a result of estate taxation. The sign of the variation of hoarders' bequests depend on savers' characteristics (degree of altruism) and not on their own characteristics (degree of altruism and preference for wealth). This is typically a general equilibrium result. The key economic variable is disposable income of the hoarders in the second period: $R_\star(\kappa)s_\star^h(\kappa)$. We know that for the hoarders, $s_\star^h(\kappa)$ decreases for low value of $\kappa$ and that $R_\star(\kappa) = (1+n)/[\gamma^M(1-\kappa)]$ increases with $\kappa$. We show that for low $\kappa$, $x_\star^{h'}(0)$ and $R'_\star(0)s_\star^h(0) + R_\star(0)s_\star^{h'}(0)$ have the same sign. The lower saving by the hoarders is more then compensated by the increase of the interest factor. This increase is particularly important when the degree of altruism of the savers is high enough.

This result is surprising in several respects. First, it does not concern the savers whose bequests always decrease. It applies to the hoarders even if they have a very low degree of altruism. Altruism and preference for wealth determine the level of bequests of the hoarders, but not how these bequests react to estate taxation. Note that all dynasties of hoarders behave identically as to an increase or a decrease of their bequests. This is quite different from what we observe in the two-class equilibrium where bequests are always negatively influenced by estate taxation.

Estate taxation affects also wealth distribution in the three-class equilibrium. It increases the share of capital held by the spenders and the hoarders at the expense of the share held by the savers. If the government wants too hurt the savers, it can introduces estate taxation. Estate taxation is clearly a questionable instrument of redistribution: it hurts the wealthy, but favors the top wealthy.

We now turn to the incidence of estate taxation on welfare[11] in the two kinds of equilibrium. Starting with the two-class equilibrium, the welfare of the spenders is:

$$V_\star^{SP}(\kappa) = (1+\beta)\ln\tilde{\omega}_\star(\kappa) + \beta\ln R_\star(\kappa) + cst$$

As $R_\star(\kappa)$ increases with $\kappa$ and the sign of $\tilde{\omega}'_\star(\kappa)$ is ambiguous, the reaction of welfare of the spenders when $\kappa$ varies is also ambiguous even for low values of $\kappa$. As to the welfare of the hoarders, we have:

$$V_\star^{HO}(\kappa) = V_\star^{SP}(\kappa) + (1+\beta)\ln\hat{\zeta}_\star(\kappa) + \delta^N \ln x_\star^{HO}(\kappa) + cst$$

The effect of estate taxation here is also ambiguous even though we know that both $\hat{\zeta}_\star(\kappa)$ and $x_\star^{HO}(\kappa)$ decreases with $\kappa$ when $\kappa$ is small.

In the three-class equilibrium, the welfare of the spenders is:

$$V_\star^{SP}(\kappa) = (1+\beta)\ln\tilde{\omega}_\star(\kappa) + \beta\ln R_\star(\kappa) + cst$$

where $\tilde{\omega}_\star(\kappa)$ decreases with $\kappa$ whereas $R_\star(\kappa)$ increases with low $\kappa$. According to Appendix E, this last effect is always dominated by the first effect at the capital stock $k_\star^M(\kappa)$. Then, a (low) increase of (low) $\kappa$ worsens the welfare of the spenders. Turning to the savers, we have:

$$V_\star^{SA}(\kappa) = (1+\beta)\ln\Omega_\star^{SA}(\kappa) + \beta\ln R_\star(\kappa) + cst$$

and we can show that a (low) increase of (low) $\kappa$ worsens the welfare of the savers. Note that the decrease of the welfare of the savers is larger than the decrease of the welfare of the spenders. We find here one of the main results of Michel and Pestieau (1998): estate taxation worsens the welfare both of the spenders and the savers. Do we find the same result for the hoarders? From (31) their welfare can be written as:

$$V_\star^{HO}(\kappa) = (1+\beta+\delta^h)\ln\tilde{\omega}_\star^h(\kappa) + \beta\ln R_\star(\kappa) - \delta^h\ln(1-\kappa) + cst$$

and we can show that:

[11] Given that $d_\star^h = \beta R_\star c_\star^h$ and $\Omega_\star^h = c_\star^h + d_\star^h/R_\star$, the long run welfare of a dynasty $h$ can be rewritten as $V_\star^h = (1+\beta)\ln\Omega_\star^h + \beta\ln R_\star + \delta^h\ln x_\star^h + cst$ where $\Omega_\star^h(\kappa) = \tilde{\omega}_\star(\kappa) + [1-\kappa-\phi_\star(\kappa)]x_\star^h(\kappa)$.

$$\left.\frac{\partial V_\star^{HO}(\kappa)}{\partial \kappa}\right|_{\kappa=0} \gtreqless 0 \quad \text{if and only if} \quad \gamma^M \gtreqless \frac{\beta[1+\alpha(\beta+\delta^h)]}{\beta[1+\alpha(\beta+\delta^h)]+\delta^h(1-\alpha)}$$

Thus if the altruism of the savers is sufficiently strong, estate taxation can have a positive effect on the hoarders, namely on bequeathers who expectedly should be penalized. This surprising result can be explained by the general equilibrium mechanism described previously that leads to increased bequests by the hoarders. However, note that the degree of altruism of the savers that is needed to increase the welfare of the hoarders is higher than that needed to increase their bequests. These increases occur when the reduction of saving by the hoarders is more than compensated by the increase in interest rate. We have seen that an increase in bequests by the hoarders does not depend on their altruism nor on their preference for wealth. The increase in utility depends (positively) on $\delta^h$.

To sum up, we have found a new reason to deal with estate taxation with caution. If the objective of such a tax is to fight top wealth holding, we have shown that society might be better off without it.

# 5 Conclusion

Traditional macroeconomic models rest on the assumption that agents are either altruistic or not and look at the shape of wealth distribution and at the effect of alternative fiscal policies on both capital accumulation and wealth distribution. Though very insightful these models fail to reflect some real life features, particularly the fact that wealth is not predominantly held by altruistic agents. Empirical studies point out to the fact that wealth accumulation, specially top wealth accumulation, is not motivated by the presence of children but by some type of preference for wealth or for the power and the prestige that wealth conveys. To incorporate this relevant and important fact, we consider in this paper agents who are characterized not only by some degree of altruism, but also by some preference for wealth. It appears that with this double heterogeneity results obtained with the sole difference in altruism don't hold true (see Table 1, 2, 3 and 4). One does not obtain necessarily the M.G.R.; in fact, overaccumulation can occur. Also, we don't have necessarily the neutrality of either public borrowing or unfunded pensions. We also show that estate tax depresses both the capital accumulation and the bequest of the altruists with no preference for wealth but not necessarily the one of the altruists with preference for wealth.

Above all our paper helps to understand why in reality top wealth is not held by the most altruistic individuals. Introduire la préférence pour la richesse nous a permis de cerner ce qu'il se passer au M.G.R. equilibrium. Si ceux sont toujours bien les most altruistic individuals qui déterminent le stock de capital de long terme de l'économie, ceux ne sont pas généralement eux qui détiennent ce capital mais les altruistes ayant une préférence pour la richesse. Then, we explain the top tail of the distribution of estates by not only altruism, as usually done, but mainly by some dynastic taste for wealth. Our finding is in the line of recent papers of Carroll (2000), De Nardi (2004) or Reiter (2004) who explain the top tail of wealth distribution in the US and/or

Sweden economy by a capitalist spirit motive according to which capital provides utility services directly, but not just through consumption. This is what we do here with our preference for wealth. Then, our paper proposed an alternative modeling of dynamic wealth distribution that is generally dealt with using calibrated versions of stochastic growth models or using theoretical models with imperfect credit market.

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