A new approach of the irreversibility effect

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Abstract

The irreversibility effect was found not to hold in more general model than in the one considered by Arrow and Fisher [1] and Henry [7]. In this paper we try to restore this irreversibility effect and we define a quasi - irreversibility effect.

1 Introduction

How soon one has to take stringent decisions in the face of a variety of uncertainties and irreversibilities?

The basic result of Arrow and Fisher [1] and Henry [7] is that there is an option value[1] to postpone irreversible investment decisions until better information is obtained. Another way to present this result is to speak of an irreversibility effect: learning reinforces the interest to take more flexible decision in the present.

In the 90's, emergence of environmental problems such as Global Warming and the discussion around the Precautionary Principle prompted economists to apply these findings in order to provide normative justifications to encourage early cautious decisions. Indeed, the Precautionary Principle is clearly an anticipatory principle which goes against "learn then act" strategies which are in practice the overwhelming behavior. The irreversibility effect is a good candidate for such normative grounds. Unfortunately, to be economically relevant to deal with problems such as global warming, a more

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[1] The standard name is the quasi - option value since the option value was already used to denote something different.

general class of decision problems than the ones considered previously in the irreversibility effect literature has to be considered. And for this larger class of models, the irreversibility effect could not be extended (see for instance Kolstad [9]). Our feeling is that this is unfortunate since the intuition for this irreversibility effect is very strong and we would have expected to find that it was a quite robust effect.

In fact, as it is convincingly put forward in Gollier and alii , it is not really the irreversibility effect that does not hold but, rather, a precautionary effect. Does the perspective of learning should make us more cautious in our present decisions? Cautious can be understood as the fact of limiting our emission of pollutants that will accumulate in the environment and that may cause negative externalities in the future. Intuitively, being cautious is a priori different from being flexible.

In this paper, we try to disentangle the role played by irreversibility from the issue of precaution and try to restore "an irreversibility effect" that we will call a *quasi irreversibility effect*. To do that, we define a notion of level of the irreversibilities in a given decision problem. Formally, we have a parameter which measures how important is the level of irreversibility constraint. Then, our main result is the following. If there is a precautionary effect for low level of irreversibilities, then there will still be a precautionary effect at higher level of irreversibilities. Let us call it a *quasi irreversibility effect.* We show that *quasi irreversibility effect* implies the irreversibility effect for the Arrow and Fisher and Henry class of models[2].

We proceed as follows. In the next section, we consider on a simple exemple the reason why an irreversibility effect may fail to hold and then we give the intution for our results. In the third section we give the formal results. The last section is devoted to a discussion.

2 A simple case

2.1 The model

Consider a simple model where one decision maker has to choose successively c_1 and c_2 and will get the following total utility level

$$U(c_1) + V\left(c_2; c_2 + \tau c_1; \theta\right)$$

[2]In a previous paper (see [3]), we presented some first results obtained in a simpler model. This previous paper focus on non bayesian decision making as well and for tractability, we had to consider a simple model. This paper is considering a standard expected utility decision maker and the results are provided in a more general model.

where θ is a state of nature which can take two values $\left\{\underline{\theta};\overline{\theta}\right\}$ with equal probability. Suppose that U and V are two strictly concave functions twice differentiable (V is twice differentiable with respect to its two first arguments).

One possible interpretation of this model is the following:

- c_1 and c_2 are some consumption level of an activity which provides some utility,
- this consumption activity produces a stock of waste $s = c_2 + \tau c_1$ with τ being the rate of decay,
- this stock of waste is provoking externalities,
- the second period utility function $V(c_2; s; \theta)$ captures simultaneously the utility of consuming and the externalities and depends on the state of nature.

Furthermore, we suppose that

$$\frac{\partial^2 V}{\partial c_2 \partial s} \leq 0$$

which means that the externalities produced by the stock of waste are more severe the higher is the level of consumption.

The seminal papers on the irreversibility effect considered the simpler case:

$$U(c_1) + V(c_2 + c_1; \theta)$$

This specification fits better an interpretation in terms of level of development or exploitation of a non renewable resource.

Irreversibility occurs because of some constraints about the choice set for c_2, that is $c_2 \in [a; +\infty[\subseteq \mathbb{R}$: consumption in the second period has to be greater than a certain level a. For instance the constraint could be that you cannot consume a negative amount. We can imagine that production constraints are such that the minimal level a is strictly positive. Conversely, the level a could be positive if some depollution device exists.

In terms of the stock of waste $s = c_2 + \tau c_1$ the minimal stock of waste is $a + \tau c_1$. Therefore, a higher c_1 restrict the possibility of monitoring the stock of waste and in this sense we say that the higher is c_1 the less flexible it is. A higher a reduces for all c_1 the choice set for the stock of waste, that is, there is more irreversibility.

In the literature that focuses on global warming or on the Precautionary Principle (see for instance Gollier and alii [6] or Ulph and Ulph [11]) an other interpretation is also proposed: to consume a higher c_1 is to be less cautious. The intuition is simply that the decision maker bequeath more waste. In the model we consider, being flexible and being cautious is the same.

2.2 Irreversibility effect and quasi - option value

The irreversibility effect states that if the decision maker expect *more information, then he should adopt a more flexible decision in the first period.* To present formally the condition under which this result is true, we consider two extreme cases of information acquisition. In the Perfect Information (PI) case, the decision maker learns the true value of θ at the end of period 1 and can choose c_2 accordingly. His optimal behavior is then to solve

$$\underset{c_1}{Max}\left[U(c_1)+\frac{1}{2}\underset{c_2\in[a;+\infty[}{Max}V\left(c_2;c_2+\tau c_1;\underline{\theta}\right)+\frac{1}{2}\underset{c_2\in[a;+\infty[}{Max}V\left(c_2;c_2+\tau c_1;\overline{\theta}\right)\right]$$

In the No Information (NI) case, he learns nothing and the decision maker optimally solve

$$\underset{c_1}{Max}\left[U(c_1)+\underset{c_2\in[a;+\infty[}{Max}\left[\frac{1}{2}V\left(c_2;c_2+\tau c_1;\underline{\theta}\right)+\frac{1}{2}V\left(c_2;c_2+\tau c_1;\overline{\theta}\right)\right]\right]$$

Suppose that all the solutions to these optimization problems are finite (and thus there are unique according to the strict concavity assumption). Note c_1^{NI} (resp. c_1^I) the optimal level of consumption in the first period in the NI case (resp. in the PI case).

An irreversibility effect holds if we observe that

$$c_1^I \leq c_1^{NI}$$

To introduce the notion of quasi -option value, let first define the value of information conditional to a decision c:

$$\begin{aligned} IV(c_1) &= \left(U(c_1)+\frac{1}{2}\underset{c_2\in[a;+\infty[}{Max}V\left(c_2;c_2+\tau c_1;\underline{\theta}\right)+\frac{1}{2}\underset{c_2\in[a;+\infty[}{Max}V\left(c_2;c_2+\tau c_1;\overline{\theta}\right)\right) \\ &\quad -\left(U(c_1)+\underset{c_2\in[a;+\infty[}{Max}\left[\frac{1}{2}V\left(c_2;c_2+\tau c_1;\underline{\theta}\right)+\frac{1}{2}V\left(c_2;c_2+\tau c_1;\overline{\theta}\right)\right]\right) \end{aligned}$$

There is a quasi -option value if

$$\frac{\partial IV}{\partial c_1} \leq 0$$

that is, *the value of information is larger the more flexible is the first period decision.* If it exists, the quasi-option value implies the irreversibility effect. Indeed, the concavity of the first period utility function implies that if $\frac{\partial IV}{\partial c_1}\left(c_1^{NI}\right) \leq 0$ then $c_1^I \leq c_1^{NI}$.

Let

$$\begin{aligned}
\underline{c_2}(a, c_1) &= \underset{c_2 \in [a;+\infty[}{Arg\max} V\left(c_2; c_2 + \tau c_1; \underline{\theta}\right) \\
\overline{c_2}(a, c_1) &= \underset{c_2 \in [a;+\infty[}{Arg\max} V\left(c_2; c_2 + \tau c_1; \overline{\theta}\right) \\
\widetilde{c_2}(a, c_1) &= \underset{c_2 \in [a;+\infty[}{Arg\max} \left[\frac{1}{2} V\left(c_2; c_2 + \tau c_1; \underline{\theta}\right) + \frac{1}{2} V\left(c_2; c_2 + \tau c_1; \overline{\theta}\right)\right]
\end{aligned}$$

W.l.o.g suppose that $\underline{c_2}(a, c_1) \leq \overline{c_2}(a, c_1)$ and since V is concave we have $\underline{c_2}(a, c_1) \leq \widetilde{c_2}(a, c_1) \leq \overline{c_2}(a, c_1)$.

There are four possible cases to contemplate depending on when (in which states) the irreversibility constraint binds in the second period.

- (i) In the first case, the constraint always binds: $\underline{c_2}(a, c_1) = \widetilde{c_2}(a, c_1) = \overline{c_2}(a, c_1) = a$. If whatever is the information obtained in the second period, it is optimal for the decision maker to choose the lowest possible consumption, then trivially the conditional information value is null: $IV(c_1) = 0$.

- (ii) In the second case, the constraint does not bind in the "good" state $\overline{\theta}$: $\underline{c_2}(a, c_1) = \widetilde{c_2}(a, c_1) = a < \overline{c_2}(a, c_1)$ and then the conditional information value is positive

$$IV(c_1) = \frac{1}{2} V\left(\overline{c_2}(a, c_1); \overline{c_2}(a, c_1) + \tau c_1; \overline{\theta}\right) - \frac{1}{2} V\left(a; a + \tau c_1; \overline{\theta}\right)$$

The first order conditions imply that $\frac{\partial V}{\partial c_2} + \frac{\partial V}{\partial s} = 0$ in $\left(\overline{c_2}; \overline{c_2} + \tau c_1; \overline{\theta}\right)$ and $\frac{\partial V}{\partial c_2} + \frac{\partial V}{\partial s} > 0$ in $\left(a; a + \tau c_1; \overline{\theta}\right)$ and thus the derivative of the information value reduces to

$$\frac{\partial IV}{\partial c_1} = \frac{1}{2}\left(\tau \frac{\partial V}{\partial s}\left(\overline{c_2}; \overline{c_2} + \tau c_1; \overline{\theta}\right) - \tau \frac{\partial V}{\partial s}\left(a; a + \tau c_1; \overline{\theta}\right)\right)$$

Note that the concavity assumption associated to the hypothesis that the second order crossed derivative is negative imply that

$$\frac{\partial V}{\partial s}\left(\overline{c_2}; \overline{c_2} + \tau c_1; \overline{\theta}\right) \leq \tau \frac{\partial V}{\partial s}\left(a; a + \tau c_1; \overline{\theta}\right)$$

and thus[3]

$$\frac{\partial IV}{\partial c_1} \leq 0$$

In the case where $V(c_2; c_2 + \tau c_1; \theta) = V c_2 + c_1; \theta$ the derivative of the information value is simply:

$$\frac{\partial IV}{\partial c_1} = -\frac{1}{2}\frac{\partial V}{\partial s}\left(a; a + c_1; \overline{\theta}\right) < 0$$

- (iii) In the third case, the constraint only binds in the "bad" state $\underline{\theta}$: $\underline{c_2}(a, c_1) = a < \widetilde{c_2}(a, c_1) \leq \overline{c_2}(a, c_1)$ and then the conditional information value is also positive

$$\begin{aligned} IV(c_1) &= \frac{1}{2}V\left(a; a + \tau c_1; \underline{\theta}\right) + \frac{1}{2}V\left(\overline{c_2}; \overline{c_2} + \tau c_1; \overline{\theta}\right) \\ &\quad -\frac{1}{2}V\left(\widetilde{c_2}; \widetilde{c_2} + \tau c_1; \underline{\theta}\right) - \frac{1}{2}V\left(\widetilde{c_2}; \widetilde{c_2} + \tau c_1; \overline{\theta}\right) \end{aligned}$$

The first order conditions imply that

$$\frac{\partial IV}{\partial c_1} = \frac{1}{2}\tau \begin{pmatrix} \frac{\partial V}{\partial s}\left(a; a + \tau c_1; \underline{\theta}\right) + \frac{\partial V}{\partial s}\left(\overline{c_2}; \overline{c_2} + \tau c_1; \overline{\theta}\right) \\ -\frac{\partial V}{\partial s}\left(\widetilde{c_2}; \widetilde{c_2} + \tau c_1; \underline{\theta}\right) - \frac{\partial V}{\partial s}\left(\widetilde{c_2}; \widetilde{c_2} + \tau c_1; \overline{\theta}\right) \end{pmatrix}$$

Our assumption imply that

$$\begin{aligned} \frac{\partial V}{\partial s}\left(a; a + \tau c_1; \underline{\theta}\right) &\geq \frac{\partial V}{\partial s}\left(\widetilde{c_2}; \widetilde{c_2} + \tau c_1; \underline{\theta}\right) \\ \frac{\partial V}{\partial s}\left(\overline{c_2}; \overline{c_2} + \tau c_1; \overline{\theta}\right) &\leq \tau\frac{\partial V}{\partial s}\left(\widetilde{c_2}; \widetilde{c_2} + \tau c_1; \overline{\theta}\right) \end{aligned}$$

Therefore, without additionnal assumption which permit to compare these derivatives in different states of nature, we cannot conclude about the sign of the derivative of the information value. However, in the case where $V(c_2; c_2 + \tau c_1; \theta) = V(c_2 + c_1; \theta)$, we have

$$\frac{\partial IV}{\partial c_1} = \frac{1}{2}\frac{\partial V}{\partial s}\left(a; a + c_1; \underline{\theta}\right) < 0$$

- (iv) In the fourth case, the constraint never binds: $a < \underline{c_2}(a, c_1) \leq \widetilde{c_2}(a, c_1) \leq \overline{c_2}(a, c_1)$ and then

$$\begin{aligned} IV(c_1) &= \frac{1}{2}V\left(\underline{c_2}; \underline{c_2} + \tau c_1; \underline{\theta}\right) + \frac{1}{2}V\left(\overline{c_2}; \overline{c_2} + \tau c_1; \overline{\theta}\right) \\ &\quad -\frac{1}{2}V\left(\widetilde{c_2}; \widetilde{c_2} + \tau c_1; \underline{\theta}\right) - \frac{1}{2}V\left(\widetilde{c_2}; \widetilde{c_2} + \tau c_1; \overline{\theta}\right) \end{aligned}$$

[3]This result corresponds to Ulph and Ulph [11] findings in their Theorem 3 (p 644) : in their model, a sufficient condition for the irreversibility effect to hold is that in the no information situation the irreversibility constraint bites.

The first order conditions implies that

$$\frac{\partial IV}{\partial c_1}=\frac{1}{2}\left(\begin{array}{c}\tau\frac{\partial V}{\partial s}\left(\underline{c_2};\underline{c_2}+\tau c_1;\underline{\theta}\right)+\tau\frac{\partial V}{\partial s}\left(\overline{c_2};\overline{c_2}+\tau c_1;\overline{\theta}\right)\\-\tau\frac{\partial V}{\partial s}\left(\widetilde{c_2};\widetilde{c_2}+\tau c_1;\underline{\theta}\right)-\frac{\partial V}{\partial s}\left(\widetilde{c_2};\widetilde{c_2}+\tau c_1;\overline{\theta}\right)\end{array}\right)$$

Just as in case (iii), the sign of the derivative of the information value is ambiguous. But in the case where $V\left(c_2;c_2+\tau c_1;\theta\right)=V\left(c_2+c_1;\theta\right)$,

$$\frac{\partial IV}{\partial c_1}=0$$

Therefore, we see that the quasi-option value result is true for the simple case where $V\left(c_2;c_2+\tau c_1;\theta\right)=V\left(c_2+c_1;\theta\right)$ but is not necessarily true for the more general case. In particular, it may fail to be true when the irreversibility contraint does not play a major role, especially in case (iv) where it does not have any binding effect. In this case, we should not say that there is no irreversibility effect: speaking of no precautionary effect is more appropriate in this case in the sense that more information does not make the decision maker more cautious. Our aim is to disentangle this precautionary effect from the effect of the irreversibilities.

2.3 The quasi-irreversibility effect

Suppose that instead of considering a given decision problem and doing some comparative static exercise with respect to information, we explore what is going on in decision problems that differs according to the level of the irreversibilities constraints. Formally, the higher a is, the more irreversibilities there is.

A first result which is quite obvious is that if a is higher, then decision maker should consume less in the first period. Indeed, consider the No Information case and let define the continuation value function as $J(c_1,a)$

$$\begin{aligned}J(a,c_1)&=\underset{c_2\in[a;+\infty[}{Max}\left[\frac{1}{2}V\left(c_2;c_2+\tau c_1;\underline{\theta}\right)+\frac{1}{2}V\left(c_2;c_2+\tau c_1;\overline{\theta}\right)\right]\\&=\frac{1}{2}V\left(\widetilde{c_2}(a,c_1);\widetilde{c_2}(a,c_1)+\tau c_1;\underline{\theta}\right)+\frac{1}{2}V\left(\widetilde{c_2}(a,c_1);\widetilde{c_2}(a,c_1)+\tau c_1;\overline{\theta}\right)\end{aligned}$$

If for a higher irrversibility level, $a'>a$, it appears that $\widetilde{c_2}(a,c_1)=\widetilde{c_2}(a',c_1)>a'$, that is, the constraint still not binds, then $\frac{\partial J}{\partial c_1}(a,c_1)=\frac{\partial J}{\partial c_1}(a',c_1)$. However, if the constraint binds at a higer irreversibility level $\widetilde{c_2}(a,c_1)\leq\widetilde{c_2}(a',c_1)=a'$ then $\frac{\partial J}{\partial c_1}(a',c_1)<\frac{\partial J}{\partial c_1}(a,c_1)$. Since the first order condition for $c_1^{NI}(a)$ is

$$U'(c_1)+\frac{\partial J}{\partial c_1}(a,c_1)=0$$

therefore $c_1^{NI}(a) \geq c_1^{NI}(a')$. In a way, the terminology "irreversibility effect" would fit perfectly for this result: *if there is more irreversibilities, then the decison maker should choose more flexibility.*

We are now going to prove informally the following result :*suppose that for an irreversibility level a, there is an irreversibility effect (in the sense given in the literature), then for a higher level a', this irreversibility effect still holds.* This is what we will call the quasi - irreversibility effect.

Suppose first that $c_1^{NI}(a)$ is such that $a < \widetilde{c_2}(a, c_1^{NI}(a))$, that is we are either in case (iii) or (iv) as considered before. Then since the constraint is not binding, then $\frac{\partial \widetilde{c_2}}{\partial a}(a, c_1^{NI}(a)) = 0$ and thus $c_1'^{NI}(a) = 0$. Therefore, in the no information case the optimal level of the first period consumption is locally invariant to a change of the irreversibility level. Using the preceeding computation, we have that:

$$\frac{\partial^2 IV}{\partial a \partial c_1} = \frac{1}{2}\tau \frac{\partial \underline{c}_2}{\partial a}(a, c_1^{NI}(a)) \begin{pmatrix} \frac{\partial^2 V}{\partial c_2 \partial s}\left(a; a + \tau c_1^{NI}(a); \underline{\theta}\right) \\ + \frac{\partial^2 V}{\partial s^2}\left(a; a + \tau c_1^{NI}(a); \underline{\theta}\right) \end{pmatrix}$$

In case (iii), it can be easily proved that $\frac{\partial \underline{c}_2}{\partial a}(a, c_1^{NI}(a)) = 1$ and

$$\frac{\partial^2 IV}{\partial a \partial c_1} = \frac{1}{2}\tau \begin{pmatrix} \frac{\partial^2 V}{\partial c_2 \partial s}\left(a; a + \tau c_1^{NI}(a); \underline{\theta}\right) \\ + \frac{\partial^2 V}{\partial s^2}\left(a; a + \tau c_1^{NI}(a); \underline{\theta}\right) \end{pmatrix} \leq 0$$

In case (iv), $\frac{\partial \underline{c}_2}{\partial a}(a, c_1^{NI}(a)) = 0$ and $\frac{\partial^2 IV}{\partial a \partial c_1} = 0$.

By assumption, there is an irreversibility effect which means that $\frac{\partial IV}{\partial c_1}(a, c_1^{NI}(a)) \leq 0$. Since $\frac{\partial^2 IV}{\partial a \partial c_1} \leq 0$, this quasi - option value will remain when increasing a.

Suppose now that we are in case (i) or (ii), that is $\underline{c}_2(a, c_1^{NI}(a)) = \widetilde{c_2}(a, c_1^{NI}(a)) = a$. It can be easily proved that

$$\begin{aligned} \frac{\partial \underline{c}_2}{\partial a}(a, c_1^{NI}(a)) + \frac{\partial \underline{c}_2}{\partial a}(a, c_1^{NI}(a)) c_1'^{NI}(a) &= \\ \frac{\partial \widetilde{c_2}}{\partial a}(a, c_1^{NI}(a)) + \frac{\partial \underline{c}_2}{\partial a}(a, c_1^{NI}(a)) c_1'^{NI}(a) &= 1 \end{aligned}$$

which means that the constraint will still bite when increasing a and thus that we will stay in case (i) or (ii). Since we know that in case (i) or (ii) there is a quasi - option value, this (informal) proof is completed.

3 Formal results

To present the formal results, we keep the utility function introduced in the previous section but we consider a more general model in terms of uncertainty and information structure. The set of states is Θ and there is a prior probability distribution π on Θ. The information structure corresponds to a set of signals $Y = \{y_i\}_{i\in I}$. Conditional to the signal y_i, the posterior probability distribution is π_i and there is a probability distribution $(q_i)_{i\in I}$ on Y satisfying

$$\pi = \sum_{i\in I} q_i \pi_i$$

Here is our main result.

Theorem 1 *(i) Consumption in the first period decreases as the level of irreversibilities increases.*

(ii) The irreversibility effect continue to hold as the level of irreversibilities increases.

The proof of (i) is straightforward. As mentioned before, result (i) is a "pure" irreversibility effect: if irreversibilities are more stringent in the future then a decision maker should take more flexible decision in order to lessen his future constraint. We propose to call result (ii) the quasi-irreversibility effect.

We can decompose the proof of (ii) in two parts. First as noted in the simple model considered before, the irreversibility effect holds whenever the constraint binds in the no information case if we assume that second order crossed derivative of the second period utility function is negative. To formalize this result, denote $c_2(a, c_1, p)$ the optimal solution of

$$\underset{c_2\in[a;+\infty[}{Max} E_p V\left(c_2; c_2 + \tau c_1; \theta\right)$$

for p a probability distribution on Θ.

Lemma 2 *If $c_2(a, c_1^{NI}(a), \pi) = a$, then there is an irreversibility effect, i.e: $c_1^{NI}(a) \geq c_1^I(a)$*

Proof. Denote

$$J(a, c_1, p) = \underset{c_2\in[a;+\infty[}{Max} E_p V\left(c_2; c_2 + \tau c_1; \theta\right)$$

Then

$$\begin{aligned} & E_{q_i} E_{\pi_i} J(a, c_1^{NI}(a), \pi_i) - J(a, c_1^{NI}(a), \pi) \\ = \; & E_{q_i} \left[\begin{array}{c} E_{\pi_i} V\left(c_2(a, c_1^{NI}(a), \pi); c_2(a, c_1^{NI}(a), \pi) + \tau c_1^{NI}(a); \theta\right) \\ -E_{\pi_i} V\left(a; a + \tau c_1^{NI}(a); \theta\right) \end{array} \right] \end{aligned}$$

Remark that for all i since, either $c_2(a, c_1^{NI}(a), \pi_i) = a$ and $\frac{\partial c_2}{\partial c_1}(a, c_1^{NI}(a), \pi_i) = 0$ or

$$E_{\pi_i}\left(\frac{\partial V}{\partial c_2} + \frac{\partial V}{\partial s}\right) = 0$$

and therefore

$$\begin{aligned}
\frac{\partial}{\partial c_1} E_{\pi_i} J(a, c_1^{NI}(a), \pi_i) &= E_{\pi_i}\left(\left(\frac{\partial V}{\partial c_2} + \frac{\partial V}{\partial s}\right)\frac{\partial c_2}{\partial c_1} + \tau\frac{\partial V}{\partial s}\right) \\
&= \tau E_{\pi_i}\frac{\partial V}{\partial s}\left(c_2(a, c_1^{NI}(a), \pi_i); c_2(a, c_1^{NI}(a), \pi_i) + \tau c_1^{NI}(a); \theta\right)
\end{aligned}$$

Thus

$$\begin{aligned}
&\frac{\partial}{\partial c_1}\left(E_{q_i} E_{\pi_i} J(a, c_1^{NI}(a), \pi_i) - J(a, c_1^{NI}(a), \pi)\right) \\
= \;& \tau E_{q_i} E_{\pi_i}\left(\begin{array}{c} \frac{\partial V}{\partial s}\left(c_2(a, c_1^{NI}(a), \pi_i); c_2(a, c_1^{NI}(a), \pi_i) + \tau c_1^{NI}(a); \theta\right) \\ -\frac{\partial V}{\partial s}\left(a; a + \tau c_1^{NI}(a); \theta\right) \end{array}\right)
\end{aligned}$$

Since for all i, $c_2(a, c_1^{NI}(a), \pi_i) \geq a$ and since $\frac{\partial^2 V}{\partial s \partial c_2}, \frac{\partial^2 V}{\partial s^2} \leq 0$, therefore

$$\frac{\partial V}{\partial s}\left(c_2(a, c_1^{NI}(a), \pi_i); c_2(a, c_1^{NI}(a), \pi_i) + \tau c_1^{NI}(a); \theta\right) \leq \frac{\partial V}{\partial s}\left(a; a + \tau c_1^{NI}(a); \theta\right)$$

and thus

$$\frac{\partial}{\partial c_1}\left(E_{q_i} E_{\pi_i} J(a, c_1^{NI}(a), \pi_i) - J(a, c_1^{NI}(a), \pi)\right) \leq 0$$

Therefore

$$\begin{aligned}
U'\left(c_1^{NI}(a)\right) + \frac{\partial J}{\partial c_1}(a, c_1^{NI}(a), \pi) &= 0 \\
&\geq U'\left(c_1^{NI}(a)\right) + \frac{\partial}{\partial c_1} E_{q_i} E_{\pi_i} J(a, c_1^{NI}(a), \pi_i)
\end{aligned}$$

and thus $c_1^{NI}(a) \geq c_1^{I}(a)$. ■

The second part of the proof rests on a different argument. Indeed, consider a situation where the constraint does not bind in the no information case, i.e: $c_2(a, c_1^{NI}(a), \pi) > a$. For low level of irreversibilities, we expect to be in such a situation. As a matter of fact, at the limit when $a \to -\infty$, the constraint will never bites. In these situation, observing a reduced consumption when the information is better has to be interpreted as a precautionary effect. What we can prove is the following : suppose that the constraint does not bind in the no information case, then if a precautionary effect occurs, then it continue to hold for higher level of irreversibilities[4]. Formally, let a^* be such that $c_2(a, c_1^{NI}(a), \pi) > a$ for all $a < a^*$.

Lemma 3 *If for a level $a \leq a^*$, the precautionary effect exists, then it exists for all level $a' \in [a, a^*)$.*

Proof. Since $c_1^{NI}(a) \geq c_1^I(a)$,

$$
\begin{aligned}
& \frac{\partial}{\partial c_1}\left(E_{q_i}E_{\pi_i}J(a, c_1^{NI}(a), \pi_i) - J(a, c_1^{NI}(a), \pi)\right) \\
= \quad & \tau E_{q_i}E_{\pi_i}\left(\begin{array}{c} \frac{\partial V}{\partial s}\left(c_2(a, c_1^{NI}(a), \pi_i); c_2(a, c_1^{NI}(a), \pi_i) + \tau c_1^{NI}(a); \theta\right) \\ -\frac{\partial V}{\partial s}\left(c_2(a, c_1^{NI}(a), \pi); c_2(a, c_1^{NI}(a), \pi) + \tau c_1^{NI}(a); \theta\right) \end{array}\right) \leq 0
\end{aligned}
$$

Since, $c_2(a, c_1^{NI}(a), \pi) > a$, $\frac{\partial c_2}{\partial a}(a, c_1^{NI}(a), \pi_i) = 0$ and thus $\frac{\partial c_1^{NI}}{\partial a} = 0$. Thus

$$
\frac{dc_2}{da}(a, c_1^{NI}(a), \pi_i) = \frac{\partial c_2}{\partial a}(a, c_1^{NI}(a), \pi_i) + \frac{\partial c_2}{\partial c_1}(a, c_1^{NI}(a), \pi_i)\frac{\partial c_1^{NI}}{\partial a} = 0
$$

Therefore

$$
\frac{d}{da}\left(\frac{\partial V}{\partial s}\left(c_2(a, c_1^{NI}(a), \pi); c_2(a, c_1^{NI}(a), \pi) + \tau c_1^{NI}(a); \theta\right)\right) = 0
$$

and thus

$$
\begin{aligned}
& \frac{d}{da}\left(\frac{\partial}{\partial c_1}\left(E_{q_i}E_{\pi_i}J(a, c_1^{NI}(a), \pi_i) - J(a, c_1^{NI}(a), \pi)\right)\right) \\
= \quad & \tau E_{q_i}E_{\pi_i}\frac{d}{da}\left(\frac{\partial V}{\partial s}\left(c_2(a, c_1^{NI}(a), \pi_i); c_2(a, c_1^{NI}(a), \pi_i) + \tau c_1^{NI}(a); \theta\right)\right) \\
= \quad & \tau E_{q_i}E_{\pi_i}\frac{dc_2}{da}(a, c_1^{NI}(a), \pi_i)\left(\frac{\partial^2 V}{\partial s \partial c_2} + \frac{\partial^2 V}{\partial s^2}\right)
\end{aligned}
$$

Note that $\frac{dc_2}{da}(a, c_1^{NI}(a), \pi_i) = 1$ if $c_2(a, c_1^{NI}(a), \pi_i) = a$ or $\frac{dc_2}{da}(a, c_1^{NI}(a), \pi_i) = 0$ otherwise. Since $\frac{\partial^2 V}{\partial s \partial c_2} + \frac{\partial^2 V}{\partial s^2} \leq 0$, therefore

$$
\frac{d}{da}\left(\frac{\partial}{\partial c_1}\left(E_{q_i}E_{\pi_i}J(a, c_1^{NI}(a), \pi_i) - J(a, c_1^{NI}(a), \pi)\right)\right) \leq 0
$$

[4] In Gollier and alii [6] model, they exhibit some sufficient condition for a precautionary effect to hold when there are no irreversibilities, that is when there is no constraint on the consumption c_2 in the second period. Then, when introducing irreversibilities, that is, a constraint of positivity for c_2 , they find that this sufficient condition was also a sufficient condition for an irreversibility effect. Our result enlightens their findings.

Thus, for all $a' \in [a, a^*)$,

$$\frac{\partial}{\partial c_1}\left(E_{q_i} E_{\pi_i} J(a', c_1^{NI}(a'), \pi_i) - J(a', c_1^{NI}(a'), \pi)\right) \leq 0$$

$c_1^{NI}(a') \left(= c_1^{NI}(a)\right) \geq c_1^I(a')$. ∎

Finally, to complete the proof of our main result, just remark that for $a \geq a^*$, we can proove that the constraint always bind in the no information case, i.e $c_2(a, c_1^{NI}(a), \pi) = a$ and that we can invoke lemma 2.

The irreversibility effect found in the Arrow- Fisher - Henry type of model is a consequence of the following corollary:

Corollary 4 *If for a level of irreversibilities a, the optimal consumption in the first period does not depend on the information structure, then there is an irreversibility effect at higher level of irreversibilities.*

Indded, when the second period utility function depends only on the stock $c_2 + \tau c_1$, for sufficiently low level of irreversibilities it will be the case that information will play no role for the first period decision. Therefore there will be an irreversibility effect for any level of irreversibilities.

4 Discussion

In this paper our first aim was to clarify in a class of decision models considered in the literature, the role of irreversibilities. In these models being flexible and being cautious was confounded and therefore an irreversibility effect is a precautionary effect and vice versa. However, the intuition of why an irreversibility effect should hold is not the same than the intuition of why a precautionary effect should hold. By playing with the irreversibility level, we prove the existence of the quasi - irreversibility effect whose effect is to enforces the precautionary effect when it exists.

What is the practical meaning of this result?

First, remark that the irreversibility level qualifies a decision problem while cautiousness or flexibility qualify a decision. The quasi - irreversibility effect is a comparative static result obtained when we make comparison between different decision problems. Loosely speaking, it says that we should observe more cautious behavior when there are more irreversibilities, or we should observe less "learn then act" strategies with higher irreversibilities. Indirectly, this interpretation gives some normative foundation to the Precautionary Principle.

In terms of cost benefit analysis, Krutilla (see [10]) put forward a nice implication of the irreversibility effect. Consider two options A and B with

B being an irreversible option. Suppose a cost benefit analysis which does not explore learning possibilities conclude that option A is the best, then a "sophisticated" cost benefit analysis which would have explore learning possibilities would have reach the same conclusion. What the irreversibility effect withdraws is a cost benefit analysis which would say: option A is the best if no learning and option B is the best if learning. What's about the quasi - irreversibility effect? The quasi - irreversibility effect says that

- when B is not an irreversible option, if the cost benefit analysis does not say that option A is the best if no learning and option B is the best if learning,
- then if B is an irreversible option, the cost benefit analysis cannot say that option A is the best if no learning and option B is the best if learning.

The quasi - irreversibility effect is less strong than the irreversibility effect but it is more often true.

Jones and Ostroy [8] gave a different interpretation for the irreversibility effect. They argue that more learning is equivalent to more uncertainties in the future. Then the irreversibility effect can be reinterpreted as : one should take a more flexible decision if the future is more uncertain. In this vein, the quasi - irreversibility effect appears to be: if without irreversibility, one take more cautious decisions when the future is more uncertain, then with irreversibility, he should continue to be more cautious when the future is more uncertain.

References

[1] ARROW, K.J., FISHER, A.C. (1974): “Environmental preservation, uncertainty and irreversibility”, *Quarterly Journal of Economics*, 88, 312-319.

[2] BLACKWELL, D. (1951): “Comparison of experiments”, in J.Neyman, ed., *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley : University of California Press, 1951), 93-102.

[3] BOUGLET, T., VERGNAUD, J.C: "Irréversibilités et Incertitude totale", mimeo, (in revision).

[4] FISHER, A. (2001) "Uncertainty, Irreversibility, and the Timing of Climate Policy", *Conference on the "Timing of Climate Change Policies"* Pew Center on Global Climate Change.

[5] FREIXAS, X., LAFFONT, J-J (1984): "On the Irreversibilty Effect", in M.Boyer, R.E. Kihlstrom (eds), *Bayesian models in Economic Theory*, Amsterdam North Holland, 1984, 149-155.

[6] GOLLIER, C., JULLIEN, B., TREICH, N. (2000): "Scientific progress and irreversibility : an economic interpretation of the Precautionary Principle", *Journal of Public Economics*, 75, 229-253.

[7] HENRY, C. (1974): "Investment decisions under uncertainty : the irreversibility effect", *American Economic Review*, 64, 1006-1012.

[8] JONES, J.M., OSTROY, R.A. (1984) "Flexibility and uncertainty", *Review of Economic Studies*, 51, 13-32.

[9] KOLSTAD, C.D. (1996): "Fundamental irreversibilities in stock externalities", *Journal of Public Economics*, 60(2), 221-234.

[10] KRUTILLA, J.V., FISHER, A.C. *The Economics of Natural Environments: Studies in the Valuation of Commodity and Amenity Resources* (Washington, D.C.: Resources for the Future, 1975).

[11] ULPH, A., ULPH, D. (1997): "Global warming, irreversibility and learning", *The Economic Journal*, 107, 636-650.