Theories and Methods of the Business Cycle.
Part 1: Dynamic Stochastic General Equilibrium Models
II. The RBC approach

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1. Introduction

- F. Kydland and E. Prescott, 1982, Econometrica, Nobel Prize in 2005.
- In the line of the Lucas critique to Keynesianism: Building a model with explicit micro-foundations taking part in the general equilibrium analysis: market clearing, no monetary factors, at odds with keynesian tradition.
- One-step forward : no rationale for macroeconomic management = the optimal growth model with short-run fluctuations induced by productivity shocks (stochastic neoclassical growth model in the line of Solow (1956), Cass (1965) and Brock-Mirman (1972)). Hard-core of the RBC approach which has been recently challenged by a lot of works.
- No longer methodological opposition between business cycle and growth research which was at the heart of the neoclassical synthesis.

- Building a successful (wrt data) business cycle model: imposing a new method based on calibration to evaluate the performance of business cycle models relative to a new definition of the business cycle facts. **Quantitative Approach**.
- The methological innovation has been criticized but is now extensively used in macroeconomics today, even by proponents of stabilization interventions. The methods initiated by Kydland and Prescott are now commonly used in monetary and international economics, public finance, labor economics, asset pricing.
- In contrast to early RBC studies, DSGE models display market failures so that government interventions are desirable.

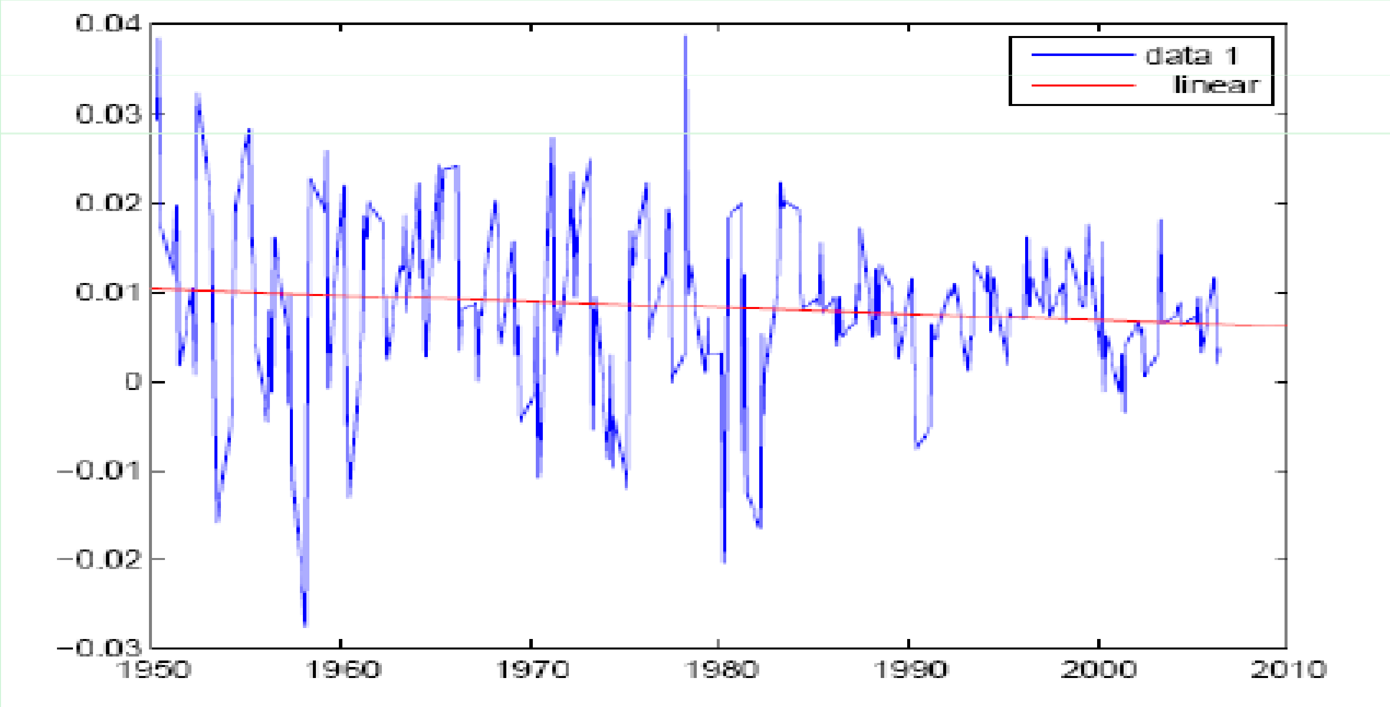
- Studying the canonical model first presented by King, Plosser and Rebelo (1988), Journal of Monetary Economics and reconsidered in King and Rebelo (1999), Handbook of macroeconomics.
- Shock-based approach : productivity shocks
- Propagated by intertemporal choices derived from dynamic optimization under rational expectations.

2. Measuring cycles

- Any time series can be decomposed as the sum of a trend and a cycle.
- Trend and cycle components are not observable. This implies to adopt a particular way of measuring them.

2.1 Growth Cycles

- Take the growth rate of the series
- Expansion: Positive rate of growth
- Problem: the cycle is very volatile



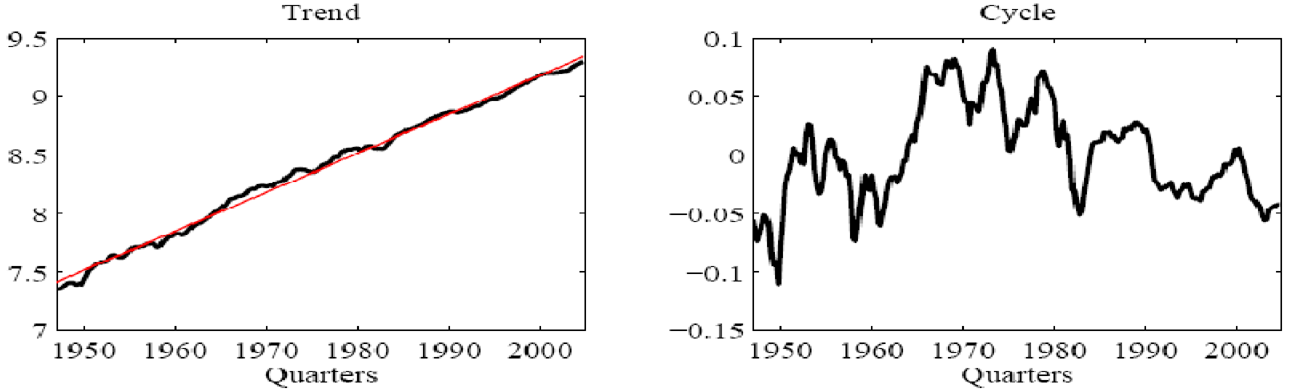
2.2 Trend Cycles

Trend Cycle

- Deviation from linear trend
- The trend is obtained from linear regression

$$\log(x_t) = \alpha + \beta t + u_t$$

- Cycle: $\widehat{x}_t = \log(x_t) - (\widehat{\alpha} + \widehat{\beta} t)$
- Expansion: Output above the trend



- Problem: the cycle can be large and very persistent (medium run fluctuations)

2.3. Measuring cycles by using Hodrick-Prescott filter

- More than identifying the non-stationarity of series, we need an economic definition of business cycles consistent with the decades of works following the seminal approach of Burns and Mitchell (NBER tradition).
- The HP filter can make stationary series up through four orders of integration.
- It is flexible enough to remove the « undesired » long-run frequencies of the stationnary component of series.
- See F. Canova [1998] for a detailed analysis of the HP filter. Journal of Monetary Economics
- See M. Baxter and R. King [1999], Review of Economics and Statistics.

- Hodrick and Prescott [1980]
- Obtained by solving

$$\min_{\{x^T_\tau\}_{\tau=1}^t} \sum_{\tau=1}^{t} (x_\tau - x^T_\tau)^2$$

subject to

$$\sum_{\tau=2}^{t-1} \left((x^T_{\tau+1} - x^T_\tau) - (x^T_\tau - x^T_{\tau-1})\right)^2 \leqslant c$$

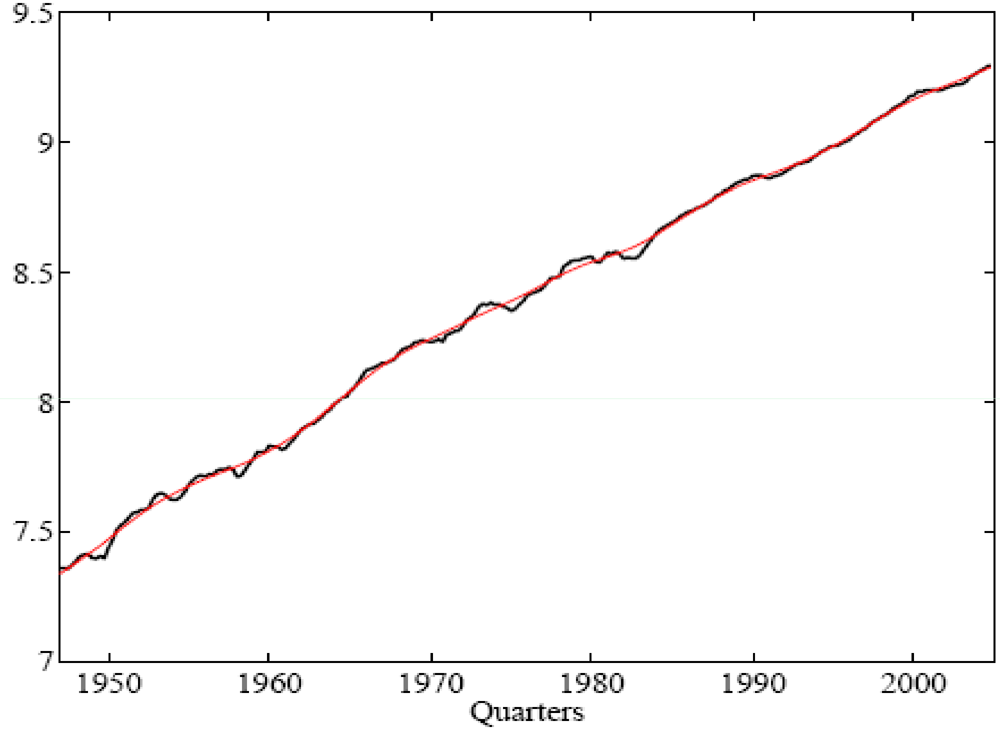
- or

$$\min_{\{x^T_\tau\}_{\tau=1}^t} \sum_{\tau=1}^{t} (x_\tau - x^T_\tau)^2 + \lambda \sum_{\tau=2}^{t-1} \left((x^T_{\tau+1} - x^T_\tau) - (x^T_\tau - x^T_{\tau-1})\right)^2$$

- $\lambda = 0$: the trend is equal to the series.
- $\lambda = \infty$: the trend is linear.

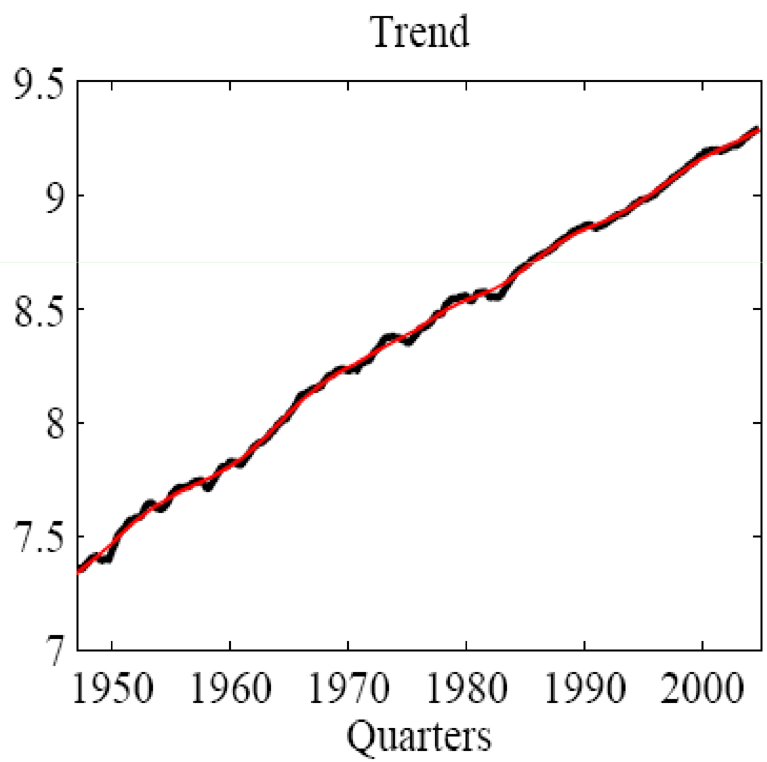
- Setting λ for quarterly data: Accept cyclical variations up to 5% per quarter, and changes in the quarterly rate of growth of 1/8% per quarter, then

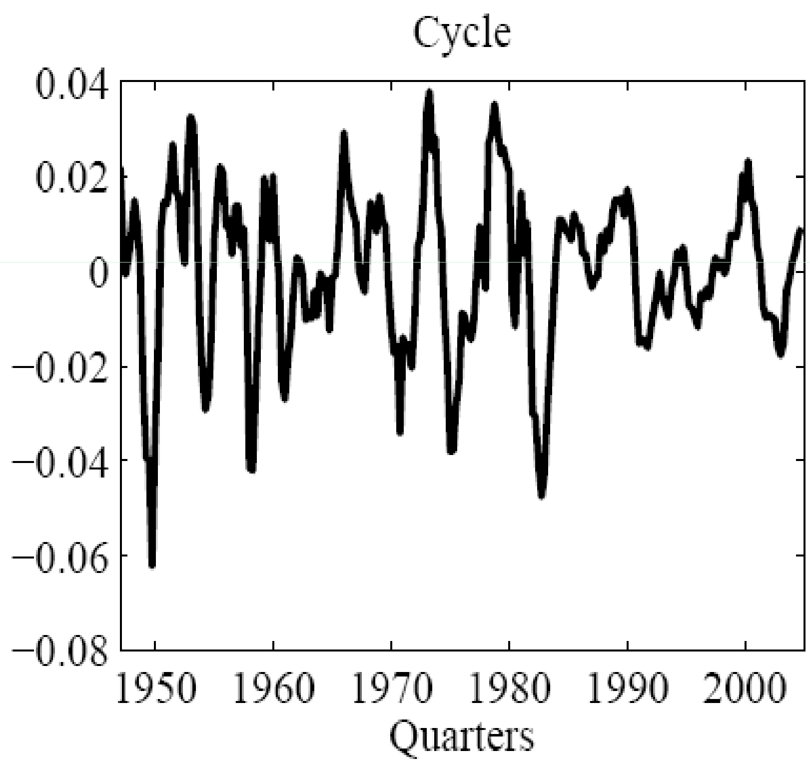
$$\lambda = \frac{5^2}{(1/8)^2} = 1600$$



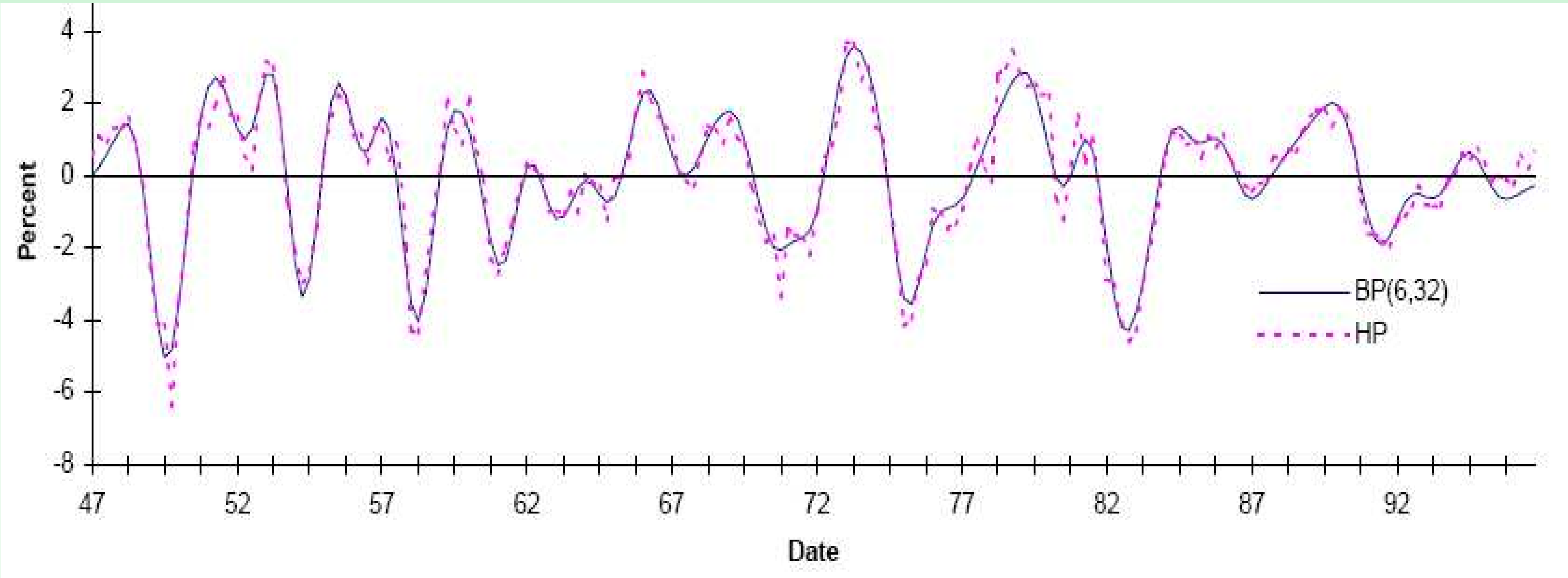
- Log of US output between 1947 and 2004
- HP trend: not linear
- cycle is the difference between the two curves

The U.S. Output Business Cycle





- To understand how HP filter works, it may be useful to compare with the measure resulting from a band-pass filter procedure: the HP filter looks like a BP filter which makes remove components of output with periodicities lower than 6 quarters and higher than 32 quarters: high frequencies like seasonnal frequencies and low frequencies are removed



3. Quantifying Business Cycles

- What are the business cycles features?
- The stylized facts that any models should aim at replicating?
- Amplitude of the cycles; Variability of macroeconomic series, differentials of variability across aggregates: compute standard deviations
- Co-variations of macroeconomic series: compute correlations
- Persistence of expansions and recessions: auto-correlation

3.1 Cyclical dynamics

Figure 2
Cyclical components of U.S. Expenditures

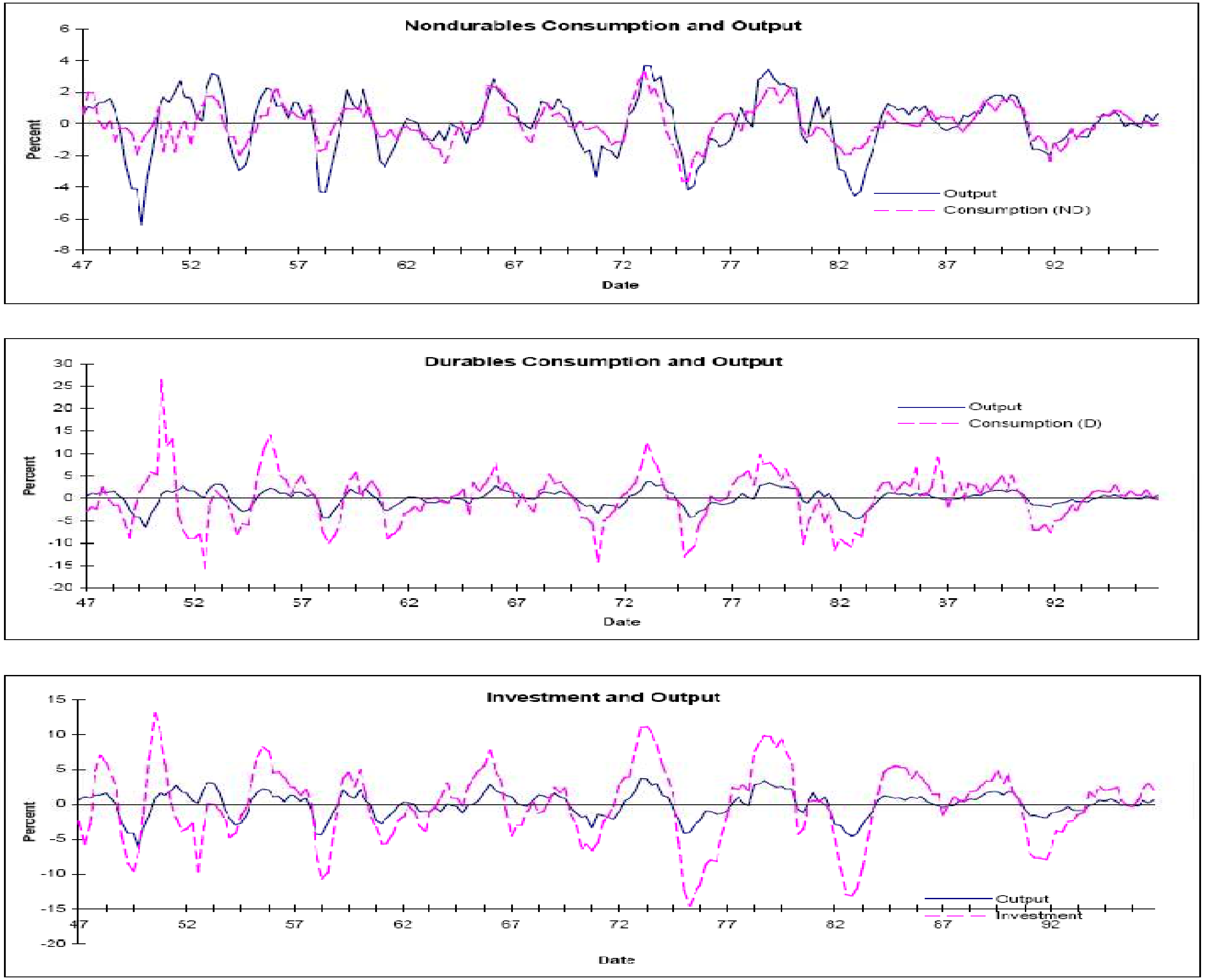
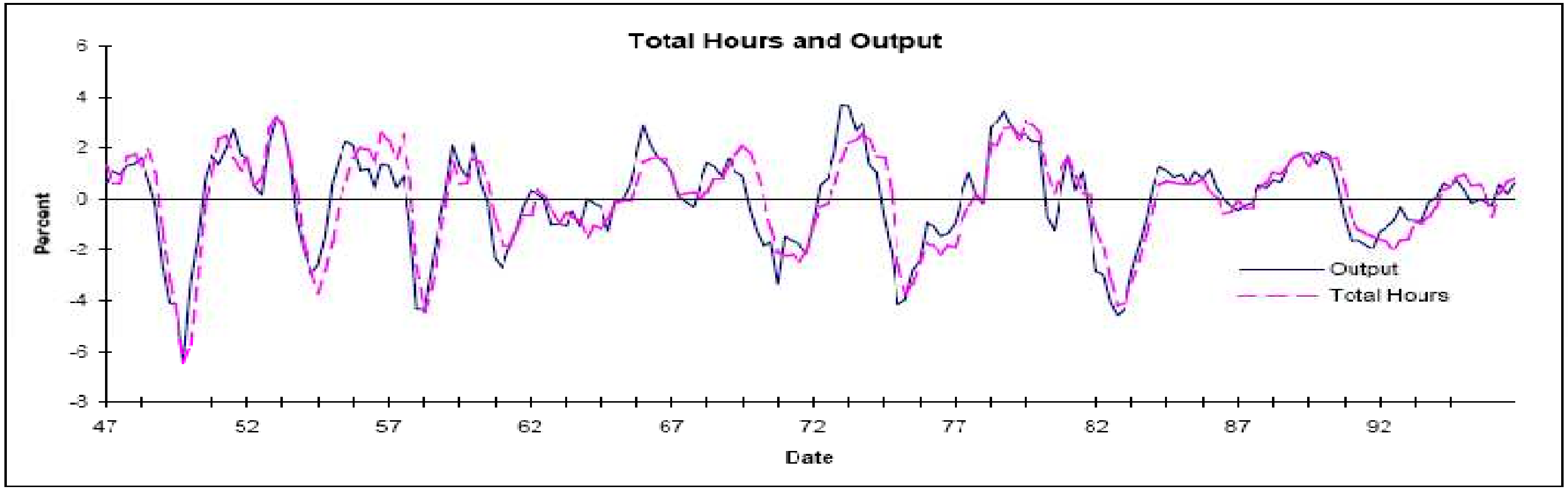


Figure 3
Cyclical component of U.S. Factors of Production



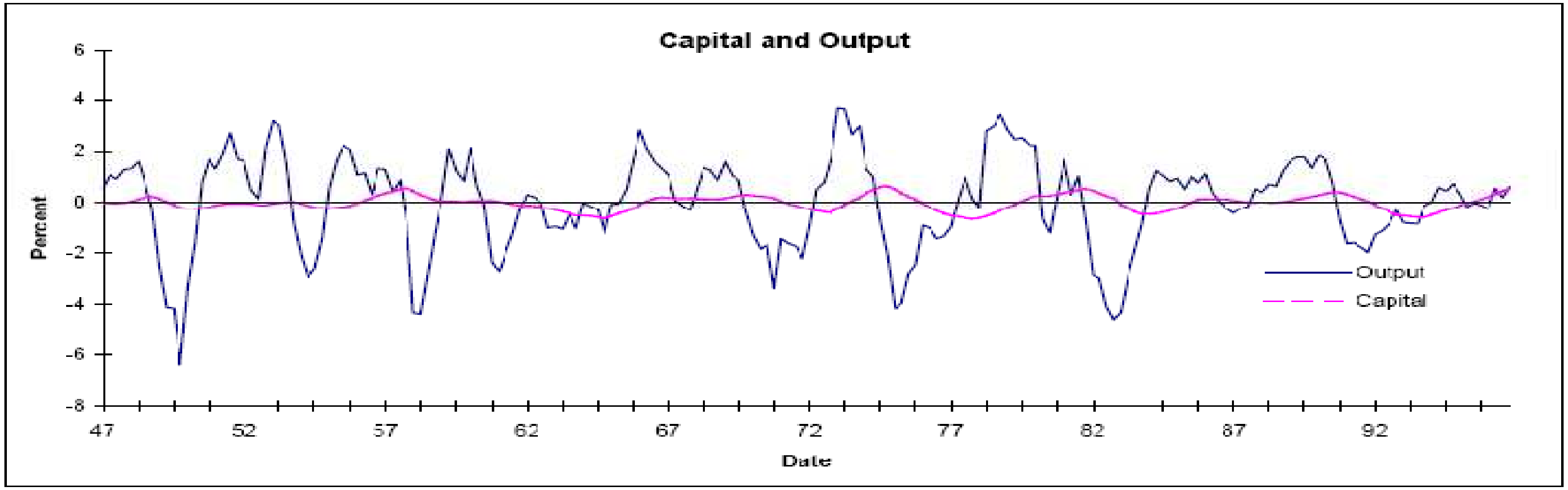
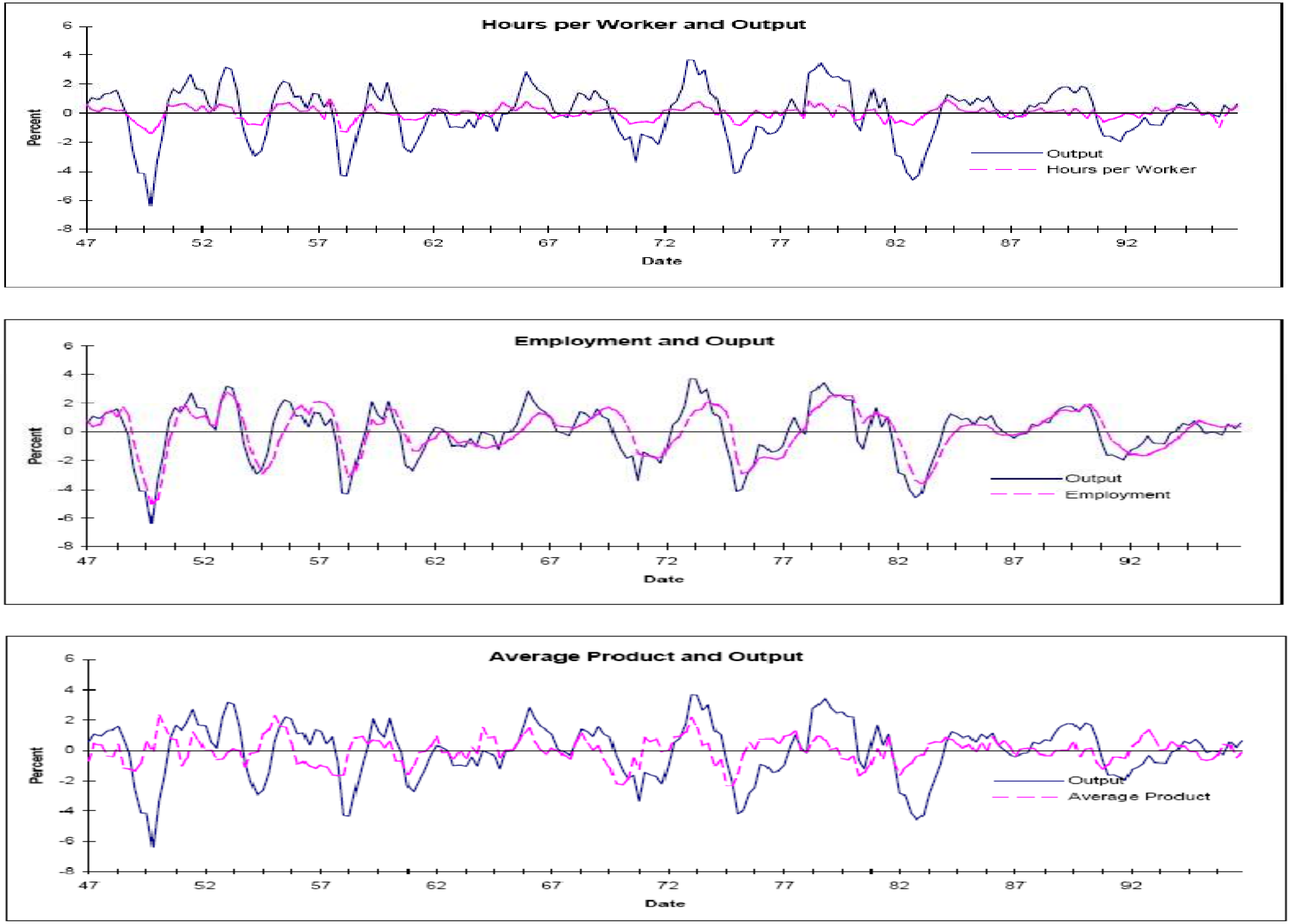


Figure 4
Cyclical component of U.S. Labor Market Measures



- Consumption of non-durables is less variable than output (panel 2-1);
- Consumer durables purchases are more variable than output (panel 2-2);
- Investment is three times more volatile than output (panel 2-3);
- Government expenditures are less volatile than output (panel 2-4);
- Total hours worked has about the same volatility as output (panel 3-1);
- Capital is much less volatile than output, but capital utilization in manufacturing is more volatile than output (panels 3-3 and 3-4)[11];
- Employment is as volatile as output, while hours per worker are much less volatile than output (panels 4-1 and 4-2), so that most of the cyclical variation in total hours worked stems from changes in employment;
- Labor productivity (output per man-hour) is less volatile than output (panel 4-3);

3.2 Quantifying Business Cycles

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	10.2	0.56	0.74	0.55

Y, C, I: per capita variables

3. 3 Are business cycles all alike?

- French Business Cycles (Hairault [1992], Economie et Prévision), 1970-1990, quarterly data. See also Danthine and Donaldson [1993], European Economic Review for an European business cycles overview.

Series	$\widehat{Y}$	$\widehat{C}$	$\widehat{I}$	$\widehat{H}$	$\widehat{P}$
Standard Deviation	.91	.81	3.64	.83	.65
Relative Std. Dev. (to $\widehat{Y}$)	1	.9	4.01	.92	.72
First Order Serial Correlation	.76	.67	.82	.89	.63
Correlation with $\widehat{Y}$	1	.63	.80	.71	.45

$\widehat{Y}$: output, $\widehat{C}$: consumption, $\widehat{I}$: investment

$\widehat{H}$: Total Hours, $\widehat{P}$: Labor Productivity

4. The canonical RBC model

- Neoclassical growth model in the line of Cass [1965]
- with stochastic productivity shocks (Brock and Mirman [1972]) and labor supply (Lucas and Rapping [1969]).
- = Time to build and aggregate fluctuations (Kydland and Prescott (1982))

See Plosser [1989], Journal of Economic Perspectives, for an enthusiastic presentation!

Many macroeconomists viewed business cycles as dead at the begining of the 70's: long expansion in the previous decade
The recession and the stagflation during the seventies challenged this view

Lucas critique

Lucas reinforced this point by arguing that microeconomic foundations frequently implied that the sorts of behavioral relations exploited by the Keynesian model builders were incapable of correctly evaluating changes in economic policy.

RBC approach

Real business cycle models view aggregate economic variables as the outcomes of the decisions made by many individual agents acting to maximize their utility subject to production possibilities and resource constraints.

explicit and firm foundation in microeconomics. More explicitly, real business cycle models ask the question: How do rational maximizing individuals respond over time to changes in the economic environment and what implications do those responses have for the equilibrium outcomes of aggregate variables?

The stochastic optimal growth model is a natural framework for the understanding of BC

benchmark model for our understanding of economic fluctuations as well as growth

What is somewhat remarkable is that the implications for fluctuations of this neoclassical approach have not been seriously explored until recently.[5]

real technological disturbances generate rich and neglected dynamics in the basic neoclassical model that appear to account for a substantial portion of observed fluctuations.

Presentation of the canonical RBC model

- King and Rebelo, Resuscitating Real Business Cycles in Handbook of Macroeconomics
- King, Plosser and Rebelo, Production, Growth and Business Cycles: I The basic Neoclassical Model, JME, 1988

4.1 Economic fundamentals

- Technology
- Preferences
- Endowments and constraints
- Initial and terminal conditions

4.1.1 Technology

Technology: The output of the economy is assumed to depend on a production function that combines labor and capital inputs.

$$Y_t = A_t F(K_t, N_t X_t), \tag{3.3}$$

- X = Deterministic component of productivity, to capture the trend in output per capita: labor augmenting technical progress

$$X_{t+1} = \gamma X_t, \qquad \gamma > 1. \tag{3.4}$$

- A = Stochastic component, to capture any transitory changes in factor productivity
- Assumed to follow a stationary process (more details later on)
- Production Function F : traditionnal regularity conditions: twice continuously differentiable, concave and homogeneous of degree one

4.1.2 Preferences

Preferences: The economy is populated by a large number of infinitely lived agents whose expected utility is defined as

$$E_0 \sum_{t=0}^{\infty} b^t u(C_t, L_t), \qquad b > 0, \tag{3.1}$$

where b denotes the discount factor, C_t represents consumption and L_t leisure.

Monentary utility u : twice continuouly diffentiable, concave.
Households like smooth consumption and leisure: in the case of transitory income shocks they will reallocate income across time: (dis)saving

❑The utility function u must be consistent to a balanced growth rate.

$$u(C,L) = \begin{cases} \frac{1}{1-\sigma}[C\nu(L)]^{1-\sigma} - \frac{1}{1-\sigma}, & \text{if } \sigma > 0, \sigma \neq 1 \\ \log(C) + \log \nu(L) & \text{if } \sigma = 1 \end{cases}.$$

❑Labor-augmenting technical progress makes the balanced steady state path feasible, but it remains to make it desirable.

❑Average consumption must grow at the constant rate given by the technical progress and average hours must be constant, whereas wages grow at the rate of the technical progress and the interest rate is constant.

❑See Appendix of King and Rebelo and King, Plosser and Rebelo in JME (1988)

4.1.2 Endowments and constraints

Endowments: The fundamental endowment that these individuals have is their time, which can be split between work (N_t) and leisure activities (L_t). Normalizing the total amount of time to one, the constraint on work is:

$$N_t + L_t = 1. \tag{3.2}$$

Abstract from other ressources (land for instance since it represents a small fraction of production factors in our modern economies)

The output of the economy can be used for consumption or investment (I_t) so that an additional resource constraint is:

$$Y_t = C_t + I_t. \qquad (3.5)$$

The stock of capital evolves according to:

$$K_{t+1} = I_t + (1 - \delta)K_t, \qquad (3.6)$$

Initial and terminal conditions

Initial conditions: The economy starts out with a capital stock $K_0 > 0$. It also begins with a level of the technology trend $X_0 > 0$, which we set equal to unity for convenience, and an initial productivity shock $A_0 > 0$.

The capital stock is the pre-determined endogenous variable.

+ terminal condition saying that the value of the capital stock must be non negative at the end (infinity!)

4.2 Stationarization of the canonical RBC model

- It is convenient to eliminate the steady state growth by detrending variables by the technical progress X. Let us denote these ratios by lower case letters: for example y=Y/X

the transformed utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, L_t), \tag{3.9}$$

with $\beta = b\gamma^{1-\sigma}$ being a modified discount factor satisfying $0 < \beta < 1$. Utility is maximized subject to the transformed constraints:

$$\begin{aligned} N_t &= 1 - L_t, && (3.10)\\ y_t &= A_t F(k_t, N_t), && (3.11)\\ y_t &= c_t + i_t, && (3.12)\\ \gamma k_{t+1} &= i_t + (1-\delta)k_t. && (3.13) \end{aligned}$$

- Slight modification of the initial economy: discount factor + capital accumulation equation.
- This is why most RBC models generally omit deterministic growth but have preferences consistent with balanced growth

4.3 Solving the planner’s problem

canonical RBC model: as if a benevolent planner maximized the welfare of the representative agent:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - N_t), \tag{A.2}$$

subject to:

$$y_t = A_t F(k_t, N_t), \tag{A.3}$$

$$y_t = c_t + i_t, \tag{A.4}$$

$$\gamma k_{t+1} = i_t + (1 - \delta) k_t, \tag{A.5}$$

$$k_0 > 0, \tag{A.6}$$

4.3.1 Deriving contingency rules and not values

- Deterministic model: the solution is a sequence of consumption, labor supply and capital accumulation

$$\{c_t\}_{t=0}^{\infty},\ \{N_t\}_{t=0}^{\infty},\ \text{and}\ \{k_t\}_{t=1}^{\infty}$$

- This solution can be made at time zero, since no relevant information is revealed later on.

- Stochastic model: random shocks are revealed over time and the solution is a set of contingency rules (decision rules or policy rules)

$$c = c(k, A)\ \text{and}\ N = N(k, A)$$

- These rules specify how much to consume and work at each point in time as a function of the state of the economy summarized by the stock of capital K and the productivity A.

4.3.2 Dynamic programming

Dynamic programming: The planner's problem can be written in recursive form as:

$$V(k, A) = \max_{c,N,k'} \{u(c, 1 - N) + \beta EV(k', A')\}, \qquad \text{(A.7)}$$

subject to:

$$c + \gamma k' - (1 - \delta)k = AF(k, N). \qquad \text{(A.8)}$$

where we use primes (′) to denote the value of a variable in the next period.

\+ $\lim_{t\to\infty} \beta^t \lambda_t k_{t+1} = 0$ terminal condition (transversality condition)

- V(k,A) : the value function of the planner's objective, ie the expected life-time utility conditionnal to k and A = the current flow of utility + the expected utility that results from starting tomorrow with k' and A' and proceeding from then on.
- k' is determined today. A' will be known only tomorrow, so we have to compute the expected value tomorrow.
- The FOC can be computed forming a Lagrangean: (A7)=objective and (A8) = constraint

4.3.3 FOCs

- Derivation with respect to c:

$$D_1 u(c, 1-N) = \lambda, \qquad \text{(A.9)}$$

λ is the multiplier associated with the constraint (A.8)

- The optimal N is given by:

$$D_2 u(c, 1-N) = \lambda A D_2 F(k, N). \qquad \text{(A.10)}$$

- The optimal k' is given by:

$$\lambda \gamma = \beta E D_1 V(k', A')$$

- The marginal utility of current consumption is equalized to the expected marginal value of capital tomorrow.
- The marginal utility of leisure is equal to the marginal product of labor times the EMVK'.
- The form of the value function is unknown. But $D_1 V(k, A)$ can be computed by differentiating the Lagrangean with respect to k.

The differentiation wrt k gives:

$$
\begin{aligned}
D_1 V(k, A) &= \lambda [A D_1 F(k, N) + (1-\delta)] \\
&+ (D_1 u(c, 1-N) - \lambda) \frac{dc}{dk} \\
&+ [\lambda A D_2 F(k, N) - D_2 u(c, 1-N)] \frac{dN}{dk} \\
&+ [\beta E D_1 V(k', A') - \lambda \gamma] \frac{dk'}{dk}.
\end{aligned}
$$

= 0 on the optimal path cf. FOC Envelop theorem

Given that c, N and k' are optimally chosen, there are zero net gains to change these values when considering a variation in k. Finally one gets:

$$D_1 V(k', A') = \lambda' [A' D_1 F(k', N') + (1-\delta)] . \quad (A.11)$$

Finally, the marginal value of capital is solution to:

$$\beta E[\lambda'(1-\delta+A'D_1F(k',N'))]=\gamma\lambda$$

- It depends on the marginal product of capital net of depreciation times the marginal value of one unit of capital tomorrow.
- The marginal product is stochastic through A': this is then the expected MPK + discounted
- Iterating forward this equation implies that the marginal value of capital is equal to the expected factor of all marginal products of capital

4.3.4 Interpreting the FOCs

- Between t and t+1, the intertemporal marginal rate of substitution for consumption is then given by:

$$\beta E_t[(A_{t+1}D_1F(k_{t+1},N_{t+1})+1-\delta)\times D_1u(c_{t+1},1-N_{t+1})]=D_1u(c_t,1-N_t)$$

- This is the so-called stochastic Euler (or Keynes-Ramsey) condition which relies the marginal rate of substitution between current and future consumptions to the marginal product of capital for a given discount factor.
- It determines the consumption rate of growth: the rate of growth of consumption is positive when the value of the MPK overcomes the discount factor ⟶ the degree of intertemporal substitution depends on the intertemporal elasticity of substitution.

- From (A9) and (A10), it is straightforward to show that:

$$D_2 u(c, 1-N) \quad = \quad D_1 u(c, 1-N) A D_2 F(k, N)$$

- The marginal rate of substitution between consumption and leisure depends on the marginal product of labor.
- It is also possible to derive the intertemporal marginal rate of substitution for leisure:

$$D_2 u(c_t, 1-N_t) = \beta E_t[A_t D_2 F(k_t, N_t) \times (A_{t+1} D_1 F(k_{t+1}, N_{t+1}) + 1 - \delta) \times \frac{D_2 u(c_{t+1}, 1-N_{t+1})}{A_{t+1} D_2 F(k_{t+1}, N_{t+1})}]$$

- The MPK relative to the discount factor determines the intertemporal substitution of leisure.
- But the MPL rate of growth plays now a crucial role: the higher is MPL tomorrow relative to MPL today, the lower is leisure today:
- MPL tomorrow gives the units of hours which can be saved by the additional output gained by working and investing more today

4.4 The decentralized economy

- Let us now consider a decentralized economy without financial assets, ie. where households accumulate the stock of physical capital.
- Households consume, work and save (accumulate capital) according to the wage rate w and the rental rate of capital (R).
- These prices are determined at the competitive equilibrium and both depend on the state of the economy, A and k.

$$w = w(\mathbf{k}, A), \quad \text{(A.16)}$$

$$R = R(\mathbf{k}, A). \quad \text{(A.17)}$$

It is useful to define the real interest rate as the rental price of capital net of depreciation:

$$r(\mathbf{k}, A) = R(\mathbf{k}, A) - \delta.$$

4.4.1 Household's dymamic programming

Household dynamic problem:

$$\nu(k_s; A, \mathbf{k}) = \max_{c, N_s, k'_s} \{u(c, 1 - N_s) + \beta E\nu(k'_s; A', \mathbf{k}')\},$$

subject to:

$$c + \gamma k'_s = w(\mathbf{k}, A)N_s + (1 + R(\mathbf{k}, A) - \delta)k_s + \pi. \qquad \text{(A.18)}$$

At the individual level, it is important to distinguish individual state variable and aggregate ones.

$$\begin{aligned} D_1 u(c, 1-N_s) &= \lambda \\ D_2 u(c, 1-N_s) &= \lambda w(k, A) \\ \beta E D_1 V(k_s'; k', A') &= \gamma \lambda \end{aligned}$$

On the optimal path, we get:

$$\begin{aligned} D_1 V(k_s; k, A) &= \lambda(1 - \delta + R(k, A)) \\ &+ (D_1 u(c, 1-N_s) - \lambda)\frac{dc}{dk_s} \\ &+ (-D_2 u(c, 1-N_s) + \lambda w(k, A))\frac{dN_s}{dk_s} \\ &+ (\beta E D_1 V(k_s'; k', A') - \gamma\lambda)\frac{dk_s'}{dk_s} \end{aligned}$$

Finally, we get the following first-order conditions:

$$\begin{aligned} D_1 u(c, 1 - N_s) &= \lambda \\ D_2 u(c, 1 - N_s) &= D_1 u(c, 1 - N_s) w(k, A) \\ \beta E[\lambda'(1 - \delta + R(k', A'))] &= \gamma\lambda \end{aligned}$$

- First condition: The present marginal utility of consumption is equalized to the marginal value of one unit of capital

- Second condition : The marginal rate of substitution between consumption and leisure is equal to the real wage.

- Third condition: the marginal value of capital today is given by the interest factor minus the depreciation rate times the marginal value of one unit of capital tomorrow (expected and discounted).

The third and the first conditions determine together the so-called stochastic Euler (or Keynes-Ramsey) condition which relies the marginal rate of substitution between current and future consumptions to the rental rate:

$$\beta E[D_1 u(c', 1 - N_s')(1 - \delta + R(k', A'))] = \gamma D_1 u(c, 1 - N_s)$$

It is also possible to show the intertemporal condition on leisure (cf. Lucas-Rapping effect):

$$\beta E[D_2 u(c', 1 - N') \frac{w(k, A)}{w(k', A')}(1 - \delta + R(k', A'))] = D_2 u(c, 1 - N)$$

4.4.2 Firm's optimization

The Firm's Problem. The firms in this economy solve a static problem. They have to decide how much capital and labor to hire in the spot markets for both of these factors:

$$\max_{k_d, N_d} \pi = AF(k_d, N_d) - wN_d - Rk_d.$$

The familiar optimization conditions for this problem are:

$$AD_1F(k_d, N_d) = R(\mathbf{k}, A), \tag{A.21}$$

$$AD_2F(k_d, N_d) = w(\mathbf{k}, A). \tag{A.22}$$

Given that the production function exhibits constant returns to scale profits will always be equal to zero:

$$\pi = AF(k_d, N_d) - AD_2F(k_d, N_d)N_d - AD_1F(k_d, N_d)k_d = 0.$$

4.4.3 Market Clearing

Market Clearing. There are three markets in this economy: spot markets for capital, labor and output. By Walras's law if two of these markets are in equilibrium the third market will also have to be in equilibrium. Thus we can state the equilibrium conditions limiting ourselves to the factor markets:

$$k_d = k_s = \mathbf{k},$$
$$N_d = N_s.$$

4.5 Equivalence betweeen equilibrium and optimum allocations

One can rearrange these conditions in order to eliminate prices:

$$\begin{aligned}\beta E[D_1 u(c', 1-N')(1-\delta + A' D_1 F(k', N'))] &= \gamma D_1 u(c, 1-N) \\ D_2 u(c, 1-N) &= D_1 u(c, 1-N) A D_2 F(k, N)\end{aligned}$$

- These conditions corresponds to the first best allocation of ressources. There is an equivalence between the optimal quantities chosen by the social planner and those in a competitive general equilibrium. Fluctuations are optimal!

Basic mechanisms

- Suppose a transitory increase in A. There will be an increase in demand for production factors. MPK and MPL have increased. W and R rise.

- The price of leisure wrt consumption increases: Ls↑ .

- As it is a transitory shock, the expected growth rate of wages is decreasing: Ls↑ through a Lucas-Rapping effect.

- As the interest rate increases, the opportunity cost of present consumption and leisure increases: more saving and more working hours.

- As there is an transitory increase in income, smoothing consumption implies to save (invest) more today.

Key parameter: Intertemporal elasticity of substitution.

It determines more especially the elasticity of working hours to productivity shocks.