

# **Theories and Methods of the Business Cycle.**

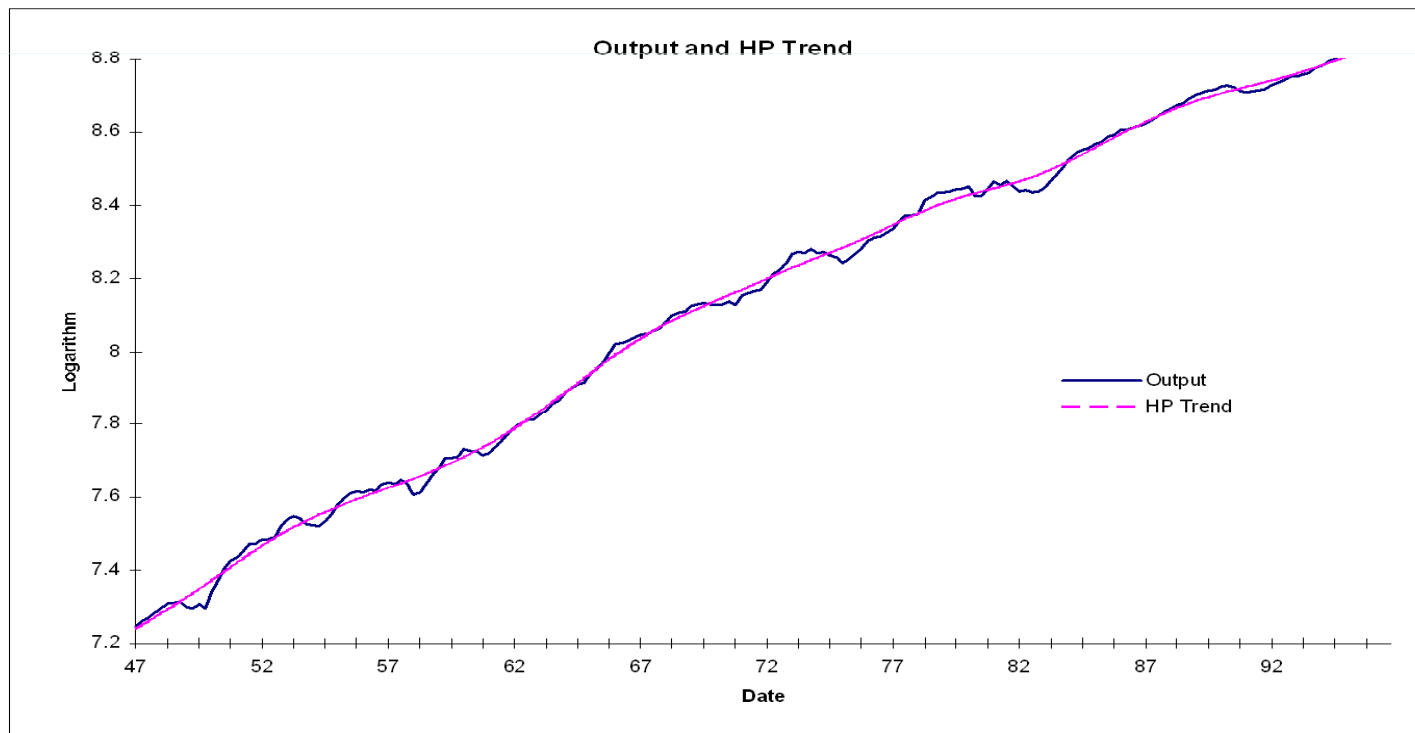
**Part I: Understanding Dynamic Stochastic  
General Equilibrium Models**  
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**Part II: Monetary and Fiscal Policy in DSGE  
Models of the Business Cycle**  
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# Introduction: Topic and Issues

- Topic: Explaining Business cycles; Short-run fluctuations around the trend, the growth path. Causes and consequences of the recurrent expansions and contractions

Figure 1  
Trend and business cycle in U.S. Real Output



# Introduction: Topic and Issues

- ❑ Several issues: methodological, applied and policy-oriented
- ❑ Part I:
  - ❑ How to define the business cycle? What are the stylized facts that any theoretical model should aim to replicating?
  - ❑ Should business cycle models be based on optimizing behavior like neoclassical growth models?
  - ❑ What are the origins of business cycles: relative contributions of real or nominal variables; fiscal and monetary?
  - ❑ Are the fluctuations optimal or the « proof » that the economic system is characterized by inefficiencies?
- ❑ Part II:
  - ❑ Is there a need to stabilize? Are there business cycles (welfare) costs?
  - ❑ What are the policies likely to stabilize the business cycle? Beyond smoothing output dynamics, stabilization policies must increase welfare.

# Overview of the course

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- ❑ I. Keynesian Cycle or Oscillator Approach : Deterministic Cycle- Stochastic Cycle-Lucas critique
- ❑ II. Real Business Cycle Approach : Methodology-Stylized Facts- The canonical model- Solving the model- Calibration- Stochastic Simulations – Beyond the canonical model (Unemployment fluctuations)
- ❑ III. Toward a new neoclassical synthesis: Money and nominal rigidities-Costs of business cycles-Optimal monetary policy

# Teaching objectives

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To be able to fully understand a dynamic stochastic general equilibrium model

- To be able to build your own model of business cycle
- In practice, choose an article (in a list available on my web page), write the computer code (with matlab) and *carry out experiments* (stochastic simulations, impulse response function) to convince the reader you provide new results wrt the literature.
- To sum up, write the computer code and around 20 pages presenting the topic and issues in a motivated introduction, the model and its solving, the experiment allowing you to put forward the main arguments and results...as in any research papers.



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# Part I: Old-Keynesian Cycle Model

# 1. Presentation of a simple model

- ❑ Reduced-Form Models without micro-foundations, ie. without explicit optimization behaviors, and backward-looking model due to adaptative expectations.
- ❑ IS-LM Tradition
- ❑ The oscillator model is a dynamic version of the IS-LM model because of an accelerator-type investment function + lagged consumption function.

$$\begin{aligned}C_t &= cY_{t-1} \\ I_t &= v\Delta^a C \\ \Delta^a C &= C_t - C_{t-1} \\ G_t &= G \\ Y_t &= C_t + I_t + G_t\end{aligned}$$

## 2. Solving the model

The output is solution of the difference equation:

$$Y_t - c(1 + v)Y_{t-1} + cvY_{t-2} = G$$

The solution is the sum of the particular solution  $Y_t = Y_{t-1} = Y_{t-2} = Y$  and of the solutions to the homogenous equation  $Y_t - c(1 + v)Y_{t-1} + cvY_{t-2} = 0$

The stationary solution is:

$$Y = \frac{G}{1 - c}$$

The solutions of the homogenous equation are:  $\alpha_1\lambda_1^t + \alpha_2\lambda_2^t$ , where  $\lambda_1$  and  $\lambda_2$  are the roots of the following polynomial equation of order 2:

$$\lambda^2 - c(1 + v)\lambda + vc = 0$$

To summarize:  $Y_t = \alpha_1\lambda_1^t + \alpha_2\lambda_2^t + Y$



## Nature of the dynamics: oscillatory or monotonic?

The output dynamics is oscillatory when the discriminant of the HE is negative, ie if  $c^2(1+v)^2 - 4cv < 0$  (complex roots).

The output dynamics is monotonic when the discriminant of the HE is non negative, ie if  $c^2(1+v)^2 - 4cv \geq 0$  (real roots).

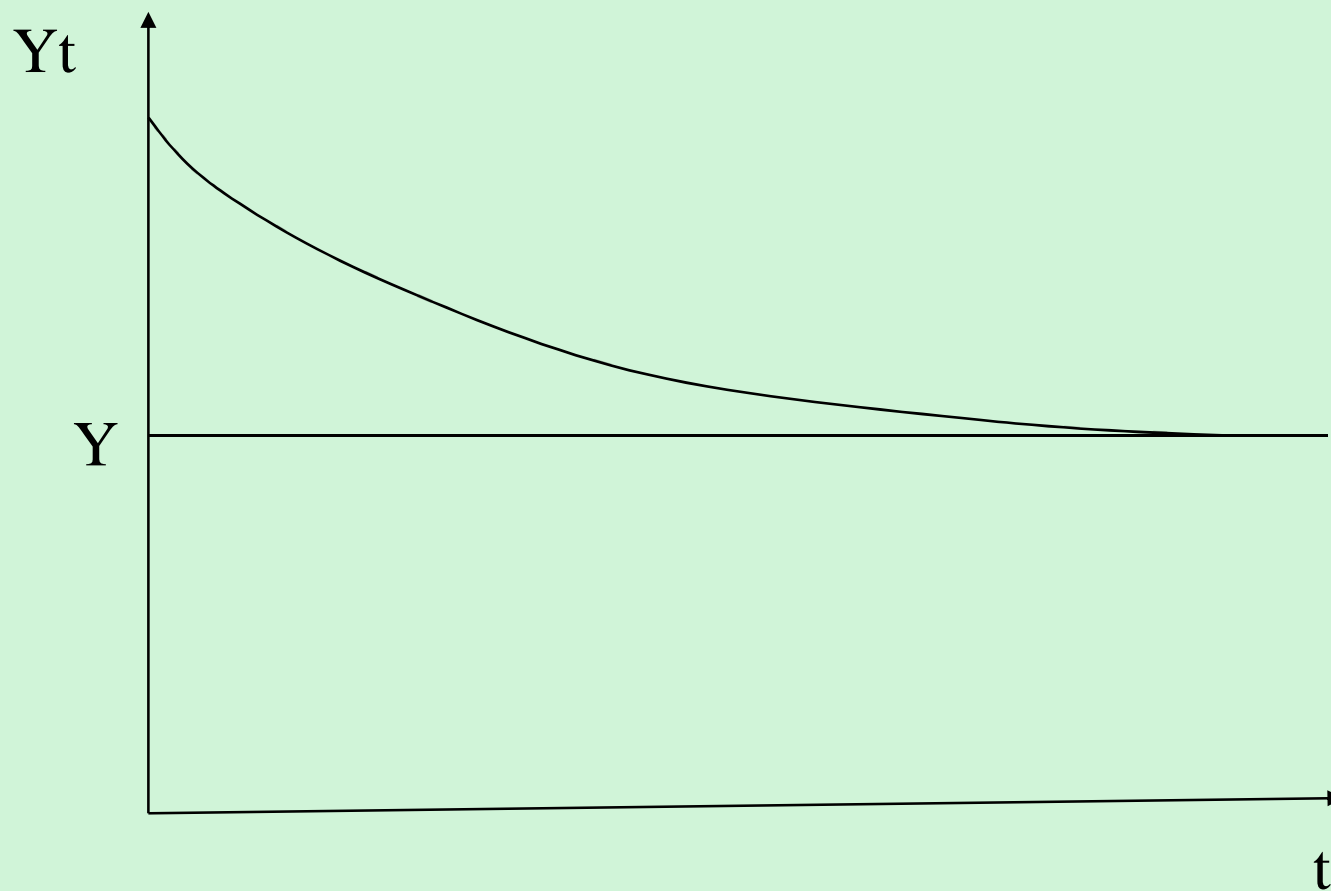
## Stability: global or local?

$Y_t$  converges to  $Y$  if  $\alpha_1 \lambda_1^t + \alpha_2 \lambda_2^t$  converges to 0.

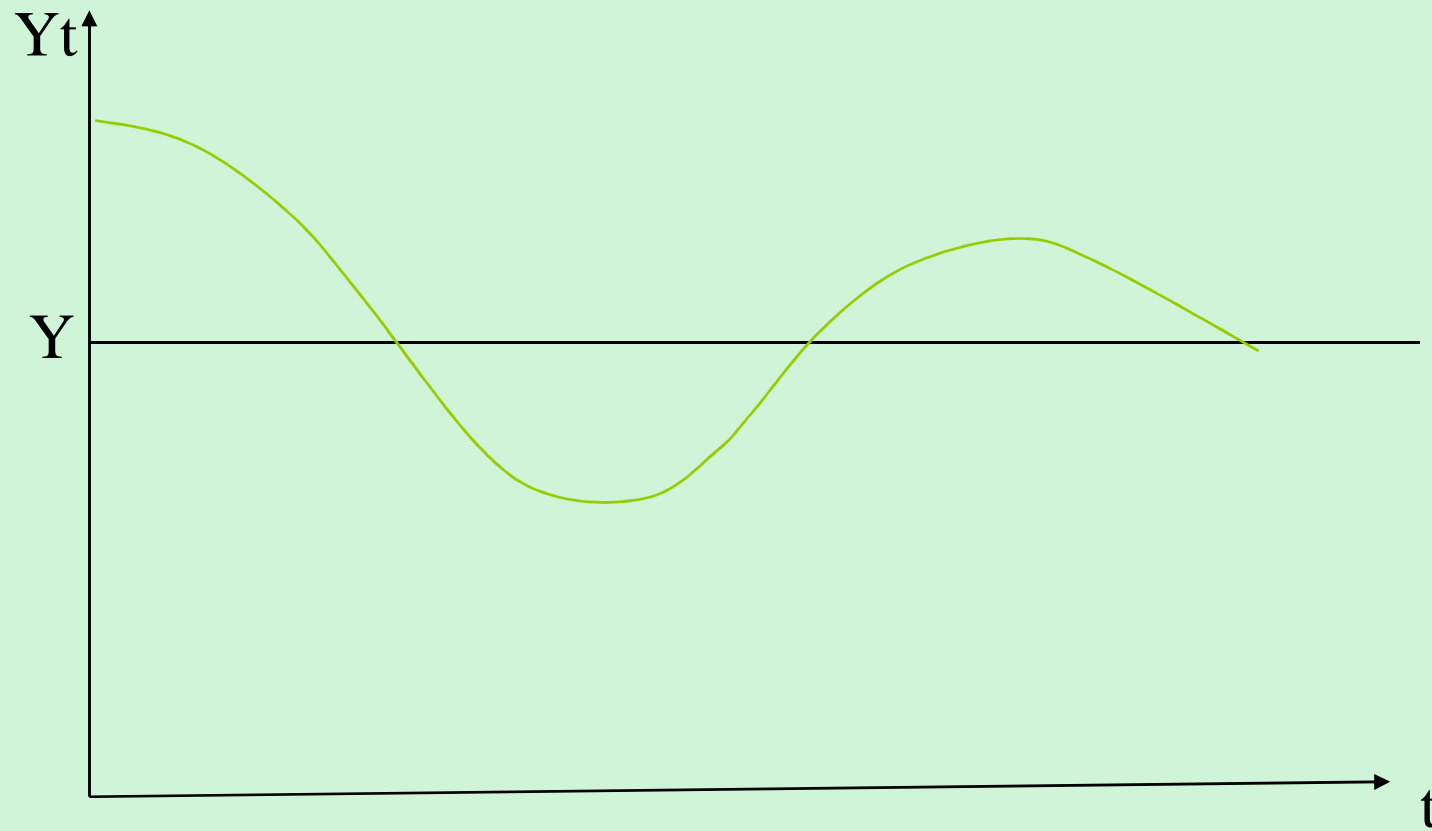
This is true whatever the values of  $\alpha_1$  and  $\alpha_2$  for values of  $\lambda_1$  and  $\lambda_2$  both inferior to 1 in absolute value. In this case, whatever the initial conditions of the system, output (and other aggregates) converges to  $Y$  (Global stability).

Let us assume that one root is superior to 1,  $|\lambda_2| > 1$ . In this case,  $Y_t \rightarrow Y$  if  $\alpha_2 = 0$ , implying a particular value for  $\alpha_1$  and a particular condition relying  $Y_0$  and  $Y_1$ . This means that there is only one trajectory converging to the stationary path (Local stability).

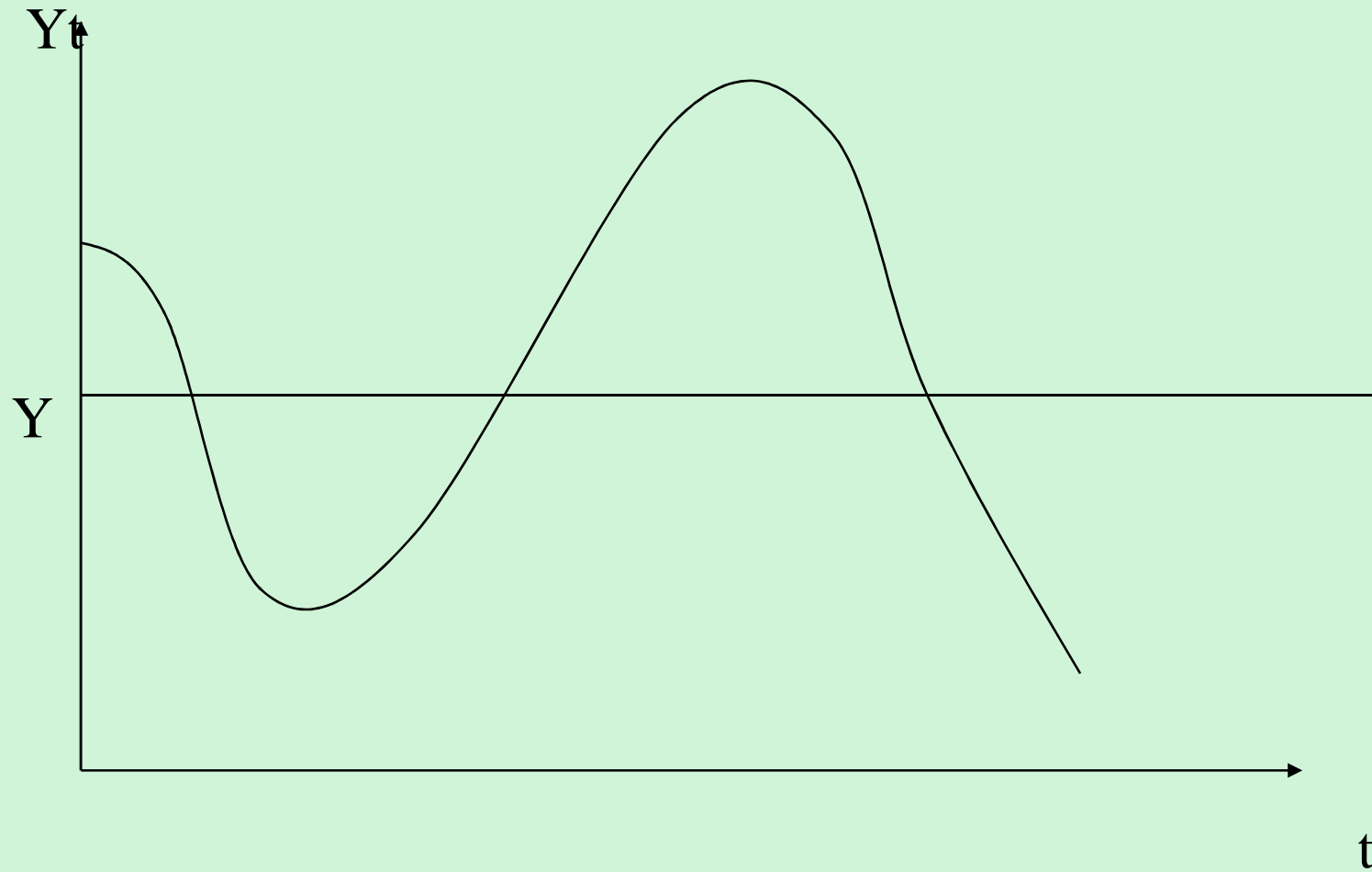
# Monotonic and stable dynamics



# Oscillatory and stable dynamics: dampened cycles

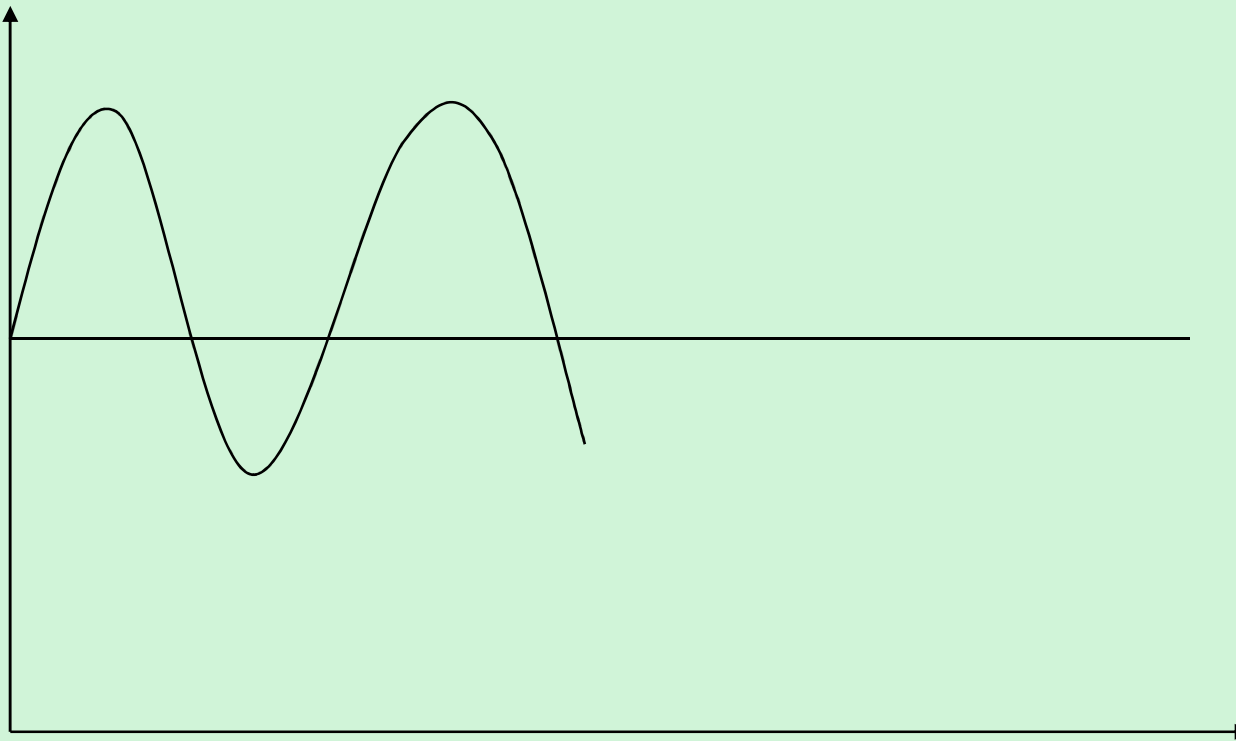


# Oscillatory and unstable dynamics: explosive cycles



### 3. Self-sustaining cycles: a very particular case...

- Cycles are permanent features only when the dynamics is oscillatory and with an unit root : Very particular values for  $c$  and  $v$ . No reasons to be verified.



## Too perfect to be realistic

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- ❑ Even if the condition holds, the cycles do not look like the observed ones: no constant periodicity and amplitude in the real world
- ❑ The deterministic framework fails to replicate observed fluctuations, even though they propose a totally self-consistent theory. This statement also applies to more recent endogenous cycle models. J.M. Grandmont (1985), *Econometrica*.

## 4. The shock-based approach

Moving to stochastic cycles in the line of Slutsky and Frisch experiment in the 1930's.

$$X_t - AX_{t-1} + BX_{t-2} = \epsilon_t$$

The shocks  $\epsilon$ , Frisch and Slutsky argued, are entirely random and distributed normally (standard variance with a mean of zero). This implies that most shocks are relatively small and approximately half of them were negative and another half positive.



## A stochastic oscillator model

- Stochastic (non predictable) shocks occur regularly and are propagated across sectors and over time by decisions taken by private agents and governments.

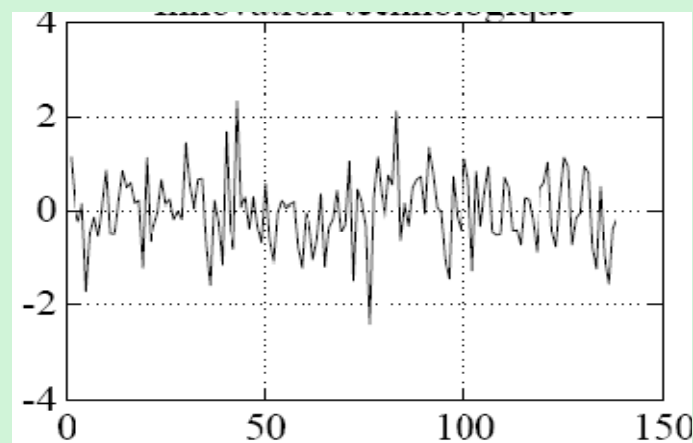
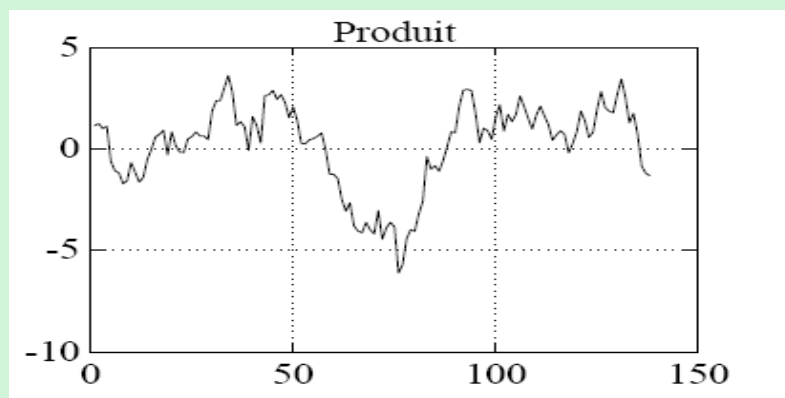
The oscillator model can be rewritten by considering a stochastic component  $\epsilon_t$  for private investment:

$$Y_t - c(1 + v)Y_{t-1} + cvY_{t-2} = G + \epsilon_t$$

- The stochastic variable is supposed to be normally distributed with a mean of zero. White noise= no serial correlation
- Stochastic simulation : To draw shocks (use *Random Number Generator*) from the assumed distribution and calculate the value of Y (iteration)

## A more realistic approach

- ❑ The dynamics is a combination of the exogenous shocks and of the internal dynamics.
- ❑ The dynamics of aggregates have no reasons to be identical to the shocks one. One can expect that the difference equation leads to « smooth » the shocks.
- ❑ The dynamics looks like cycles that are not « perfect » but looks like the observed one.



## 4. The Lucas critique

Can the oscillator model be used to experiment the impact of new government policies?

Changing the rule followed by the government expenditures ( $G$  to  $G'$ ) leads to a new dynamic path for the output:

$$Y_t - c(1 + v)Y_{t-1} + cvY_{t-2} = G'$$

It is then tempting to answer YES to the question.

However the predicted impact depends on the estimation of the parameter  $c$  and  $v$  entering into consumption and investment equations.

For Lucas (1976), these equations are not structural and the parameters they contain are not deep parameters, ie. independent from the policy followed over the past.

In other words, the equations of consumption and investment combine parameters relative to preferences and technology and parameters relative to the government decisions.

These equations cannot be considered as invariant to policy changes.  
Needs of micro-foundations.